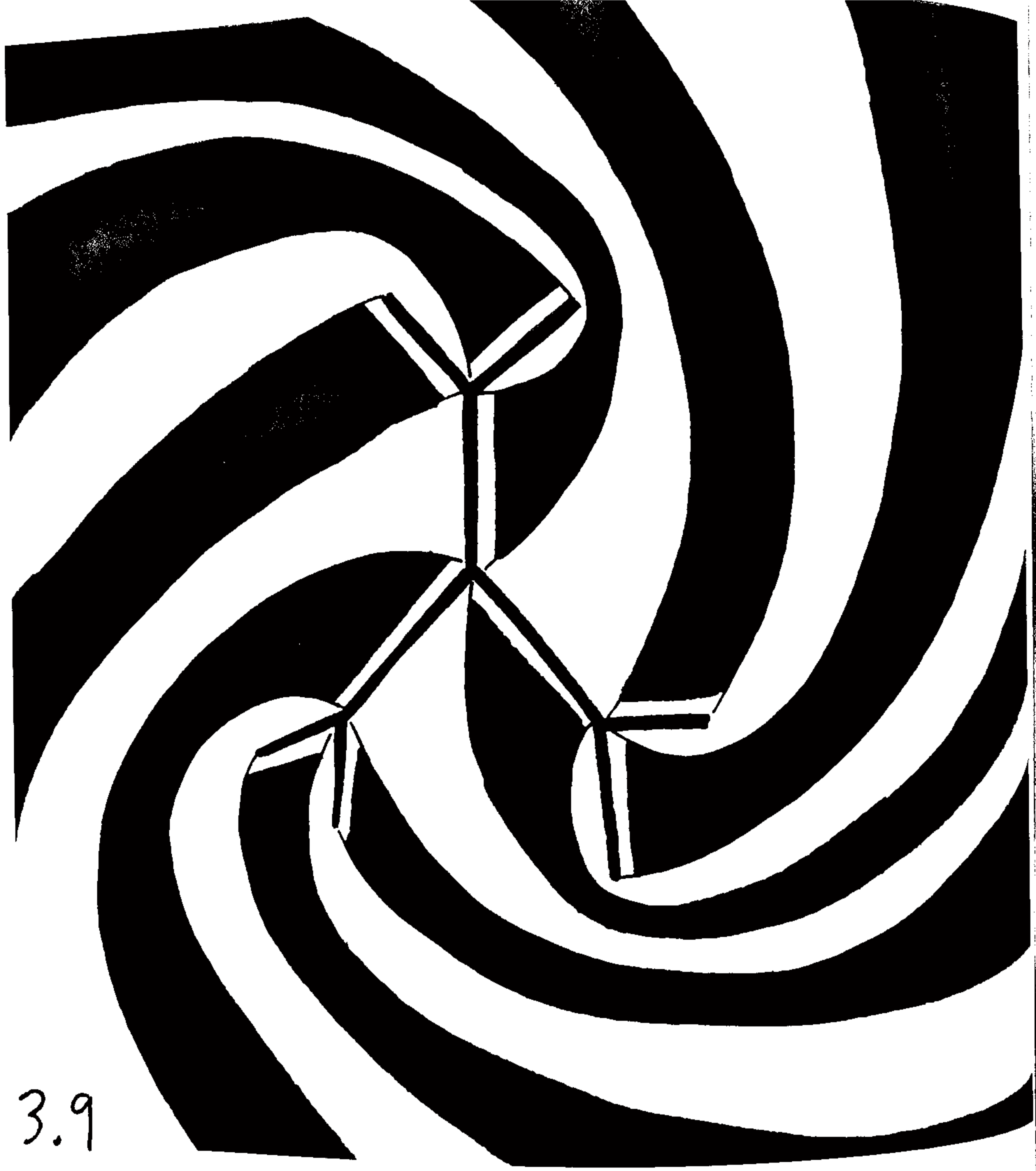
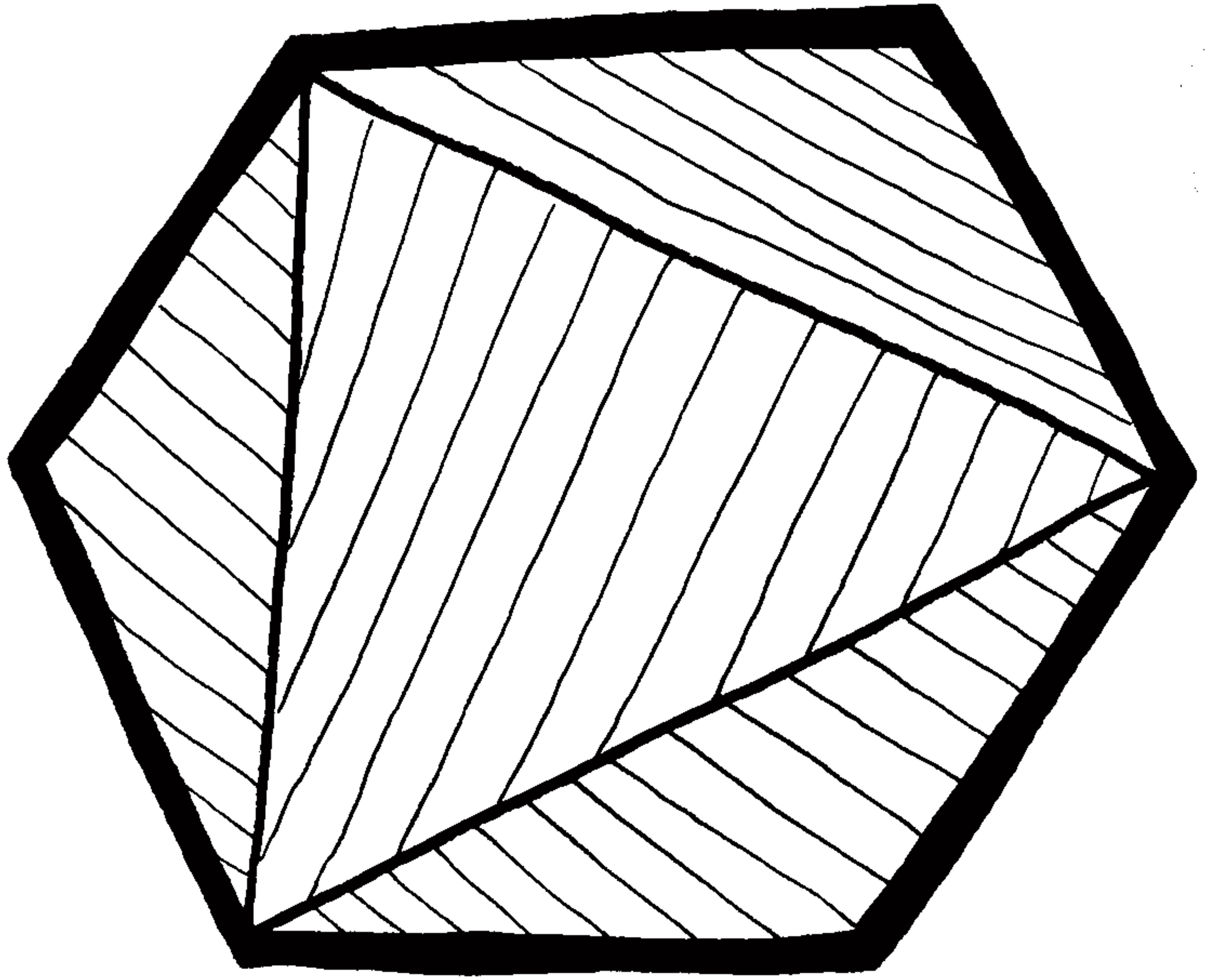


3.9

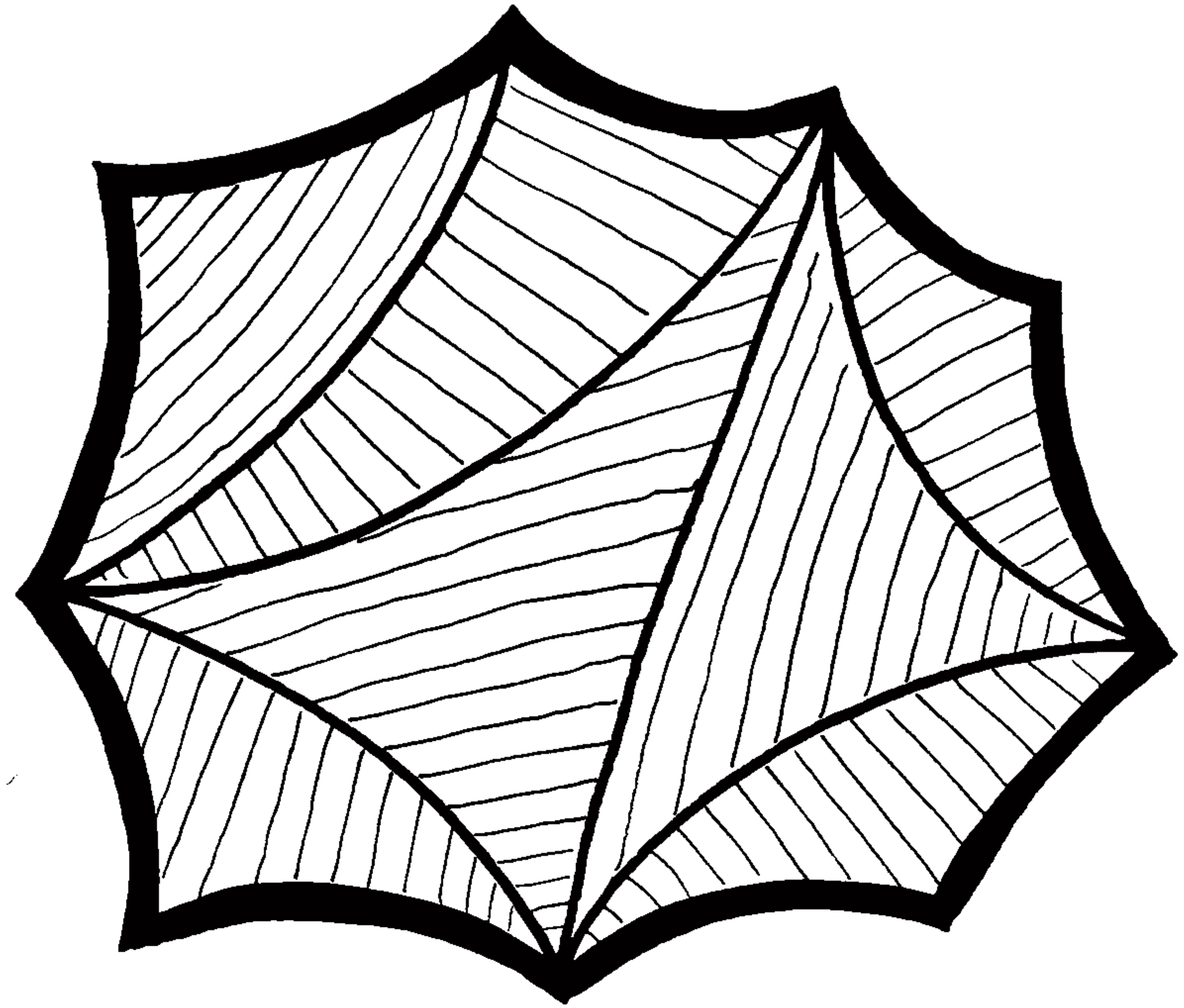
4

Trees and Polygons

Here is a polygon divided into triangles.

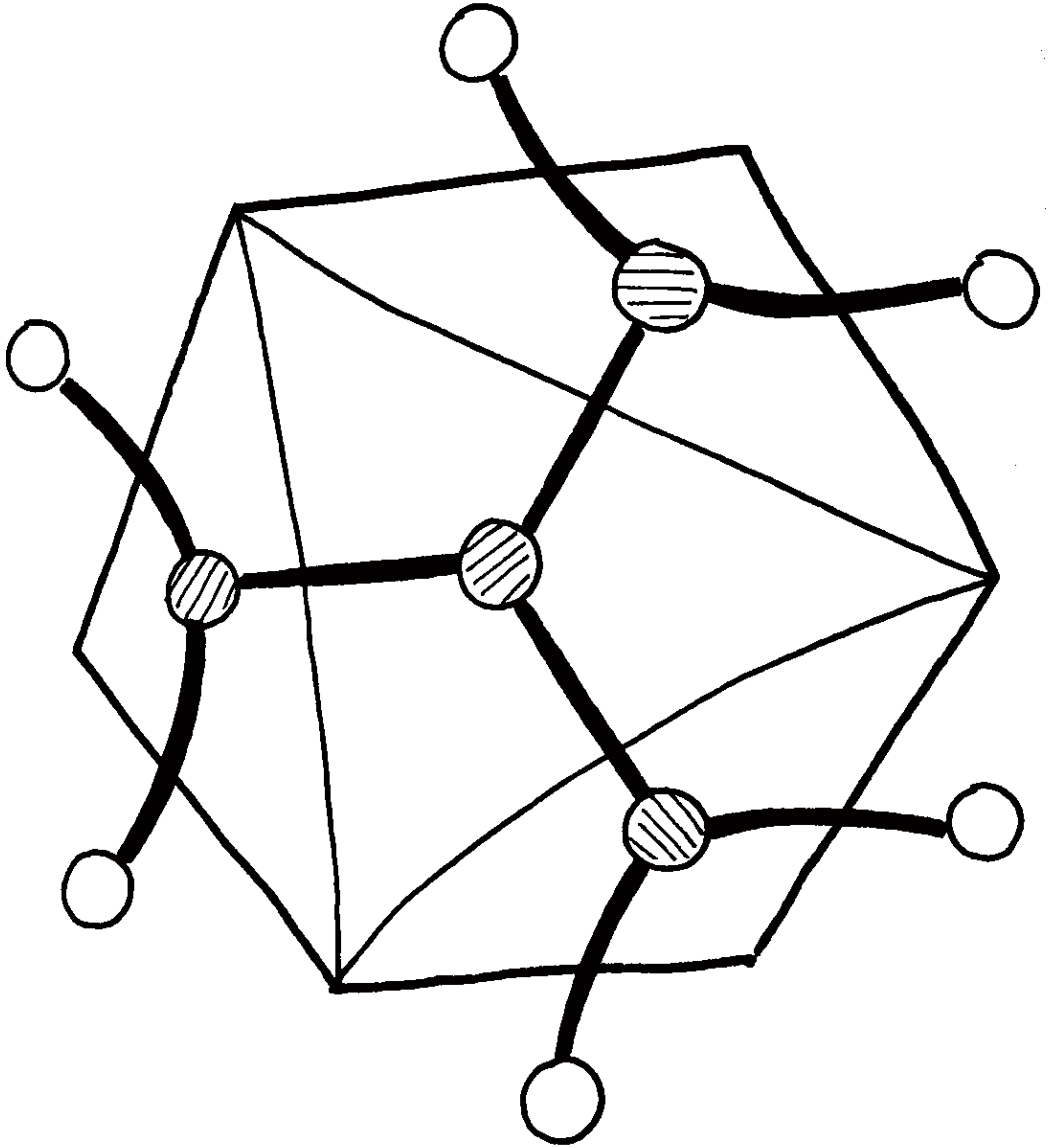


Here is another.

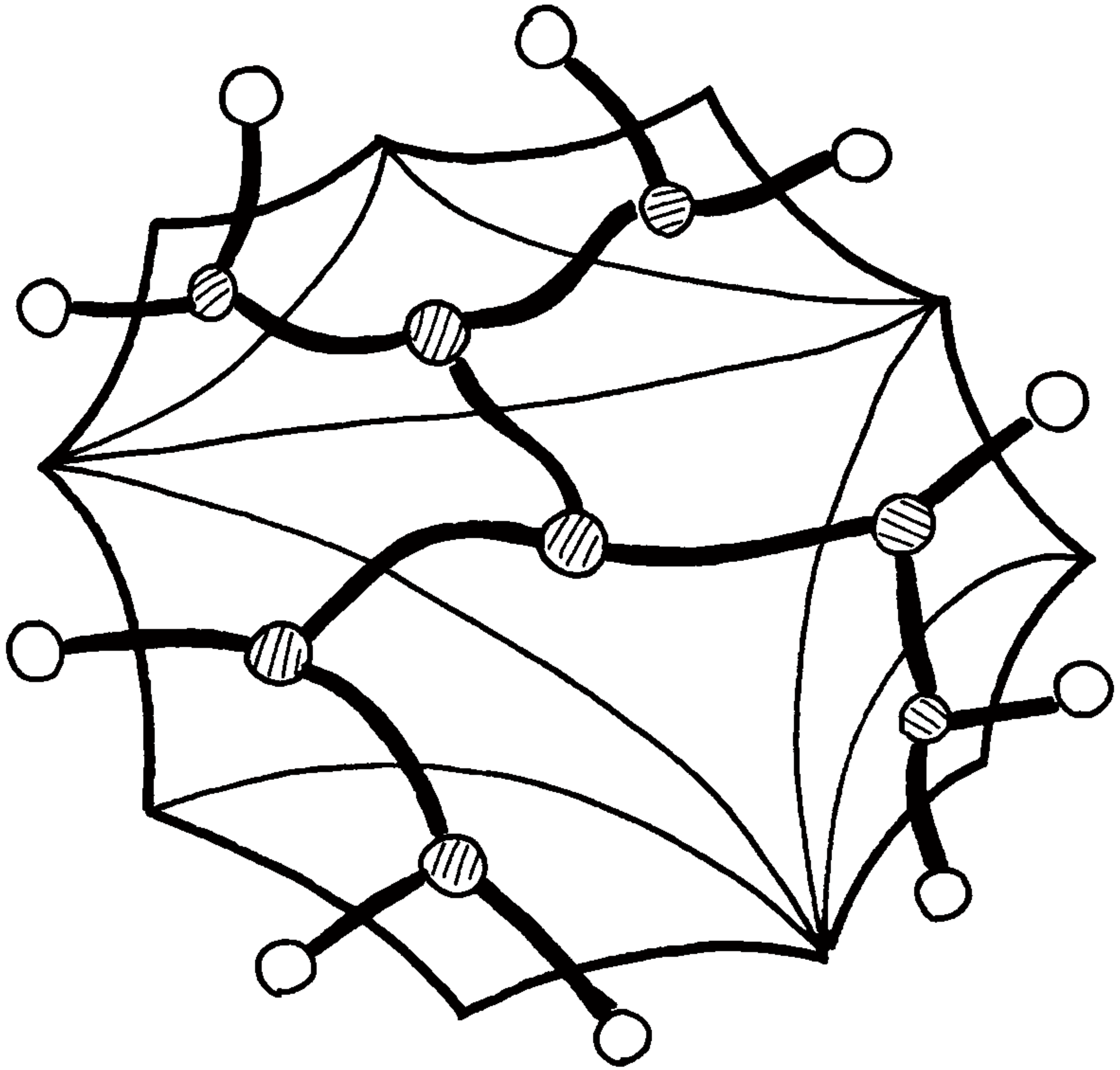


I like to curve the edges to make them more dramatic.

Trees encode the pattern.

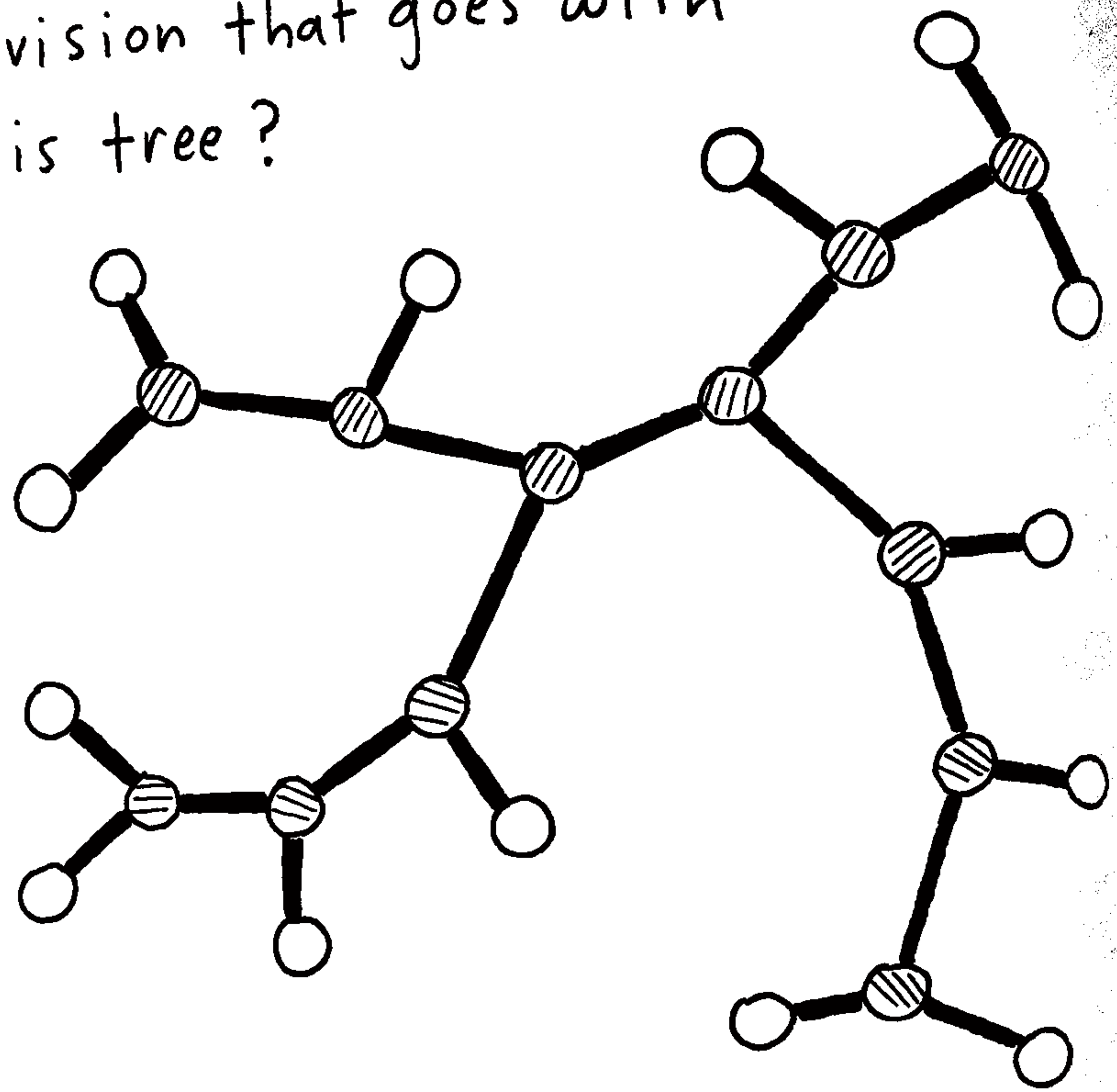


4.3



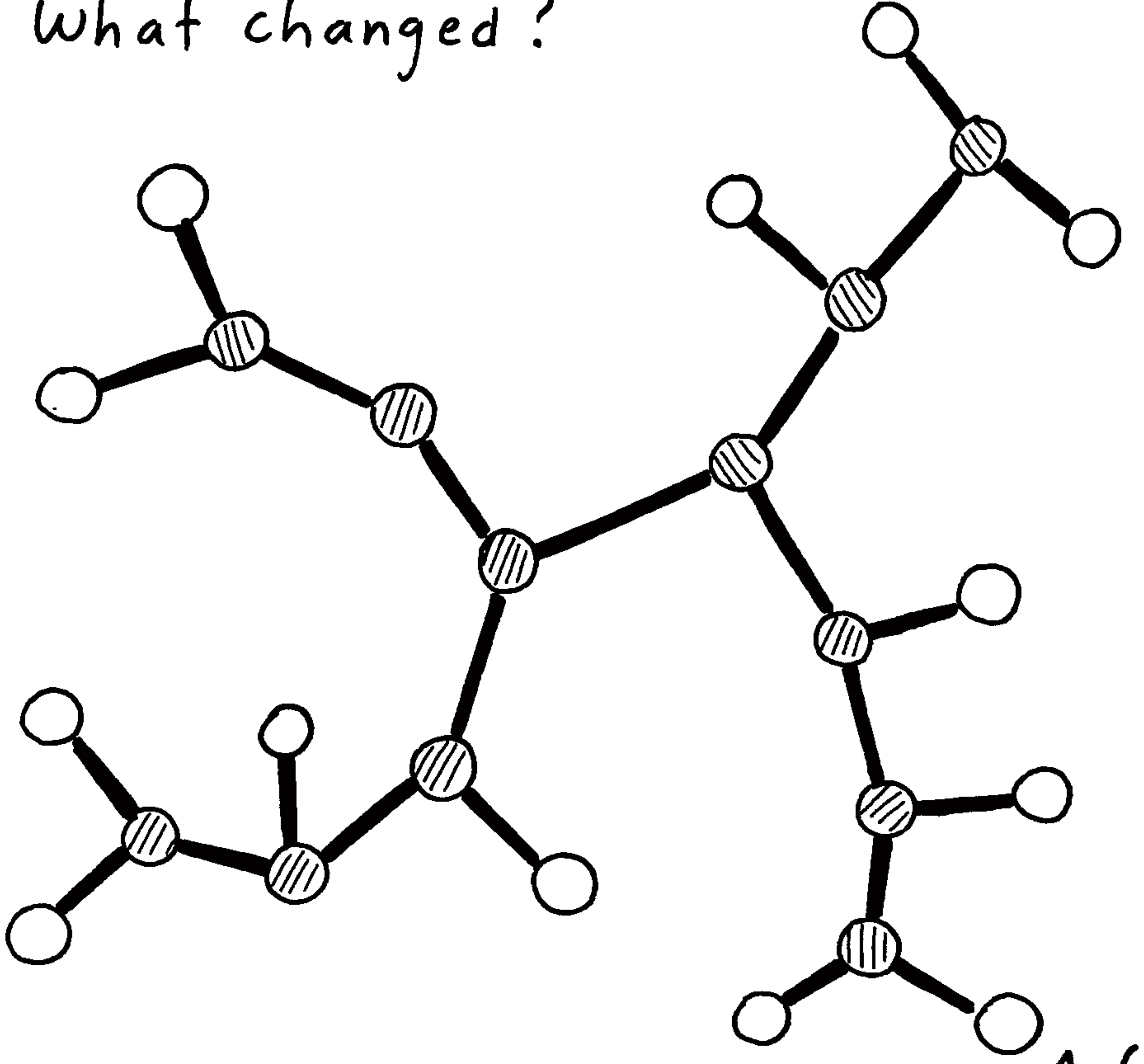
4.4

Can you draw a triangle division that goes with this tree?

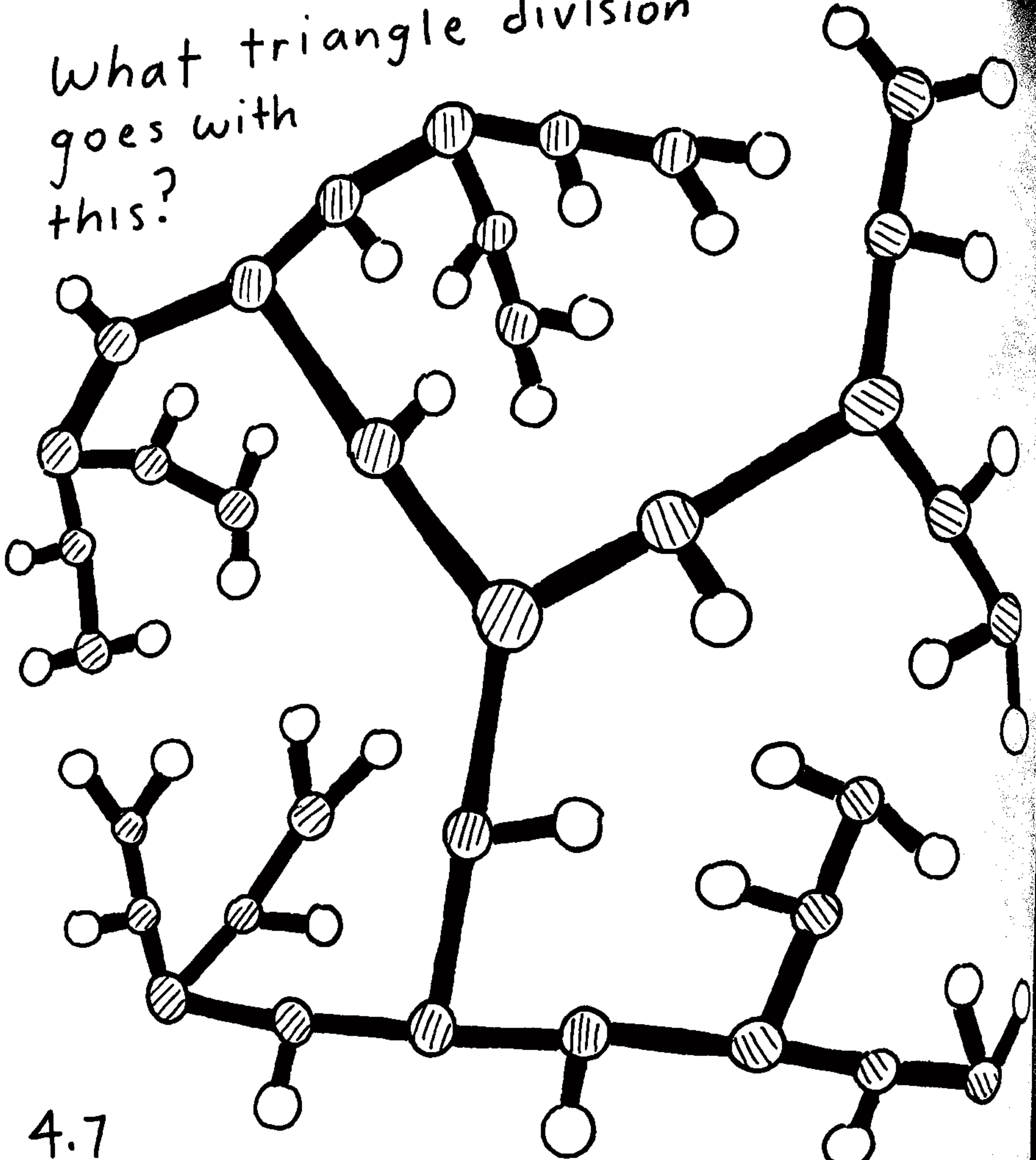


You can move the tree around.

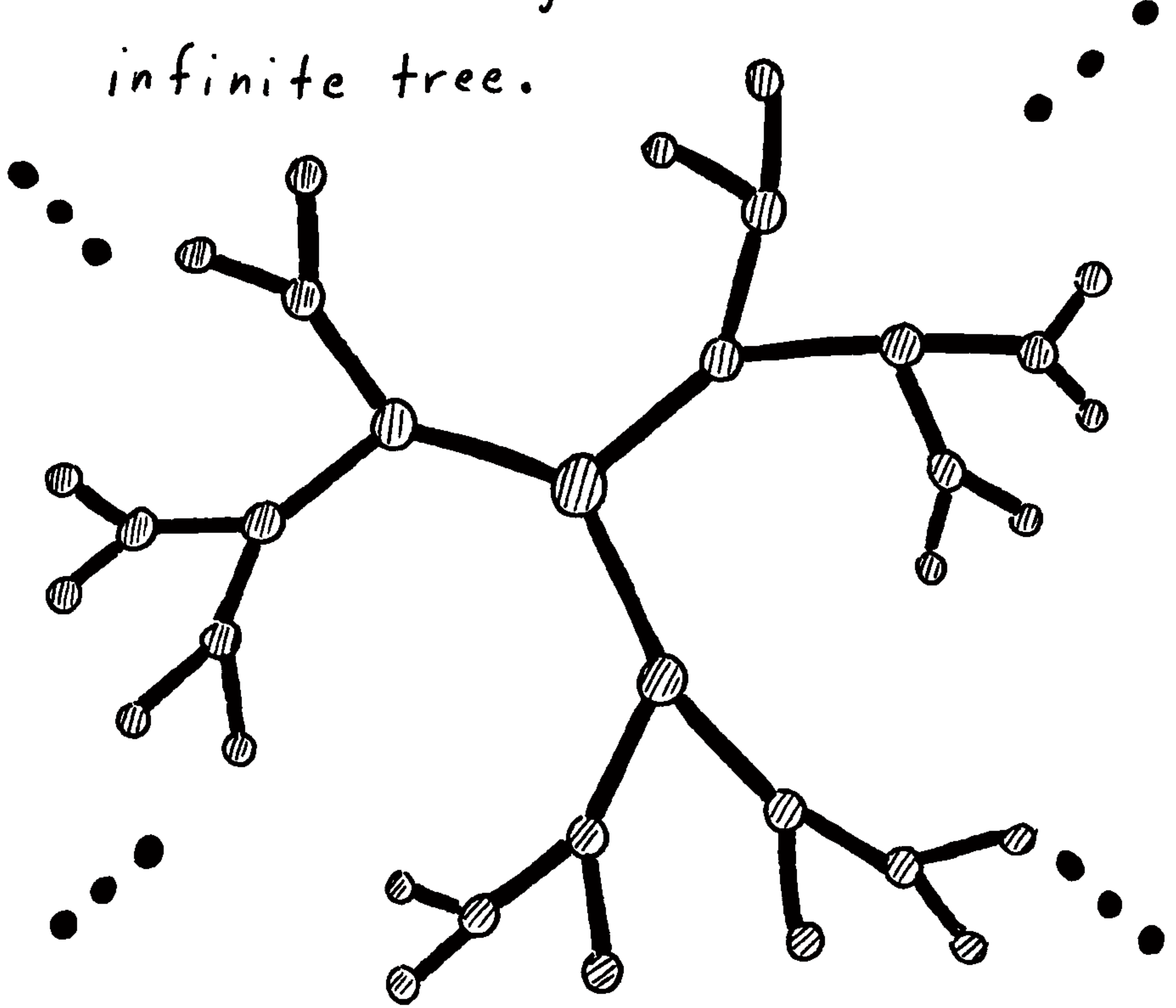
What about this one? What changed?

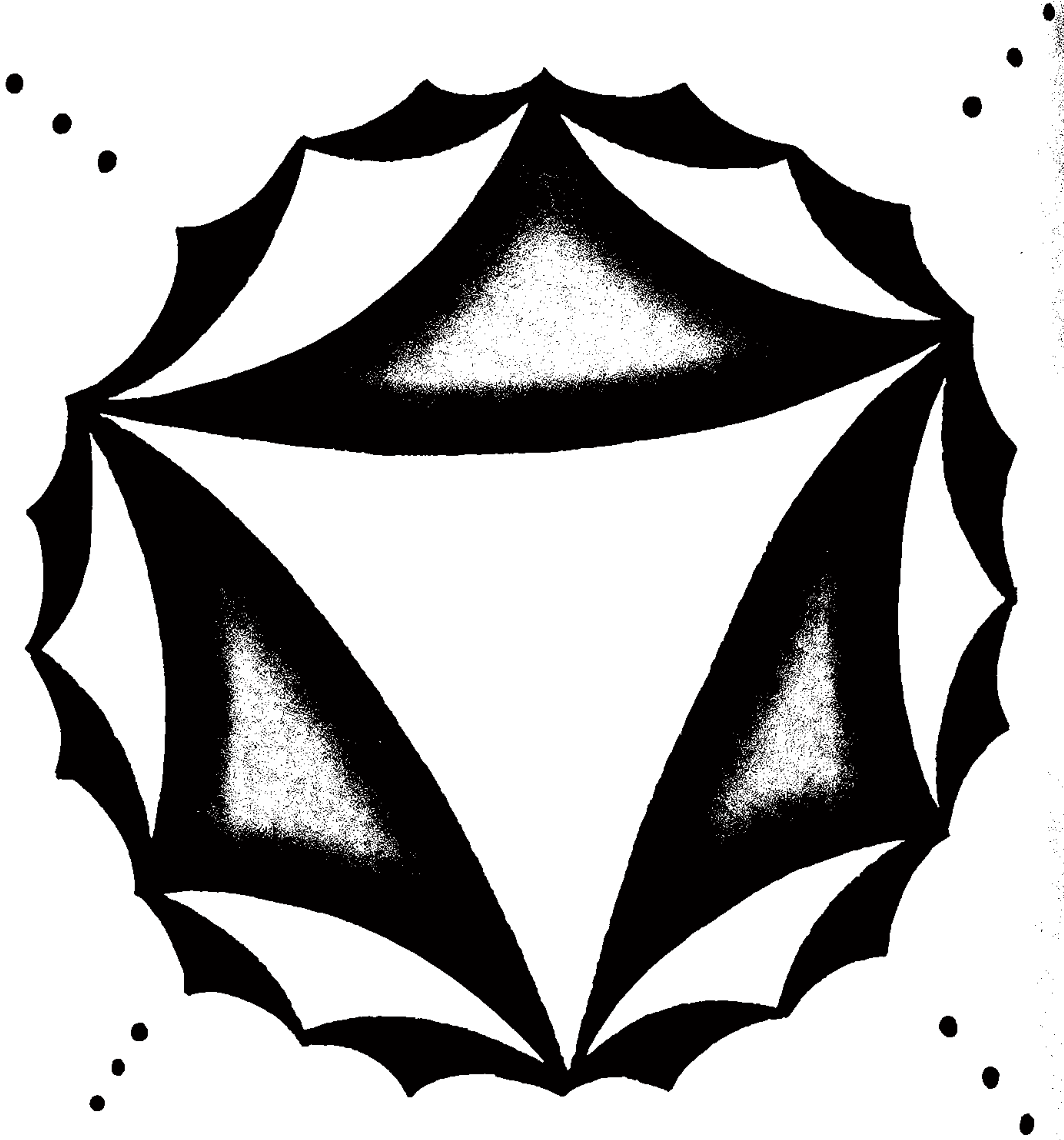


What triangle division goes with this?



I like to think about the triangle division that goes with this infinite tree.



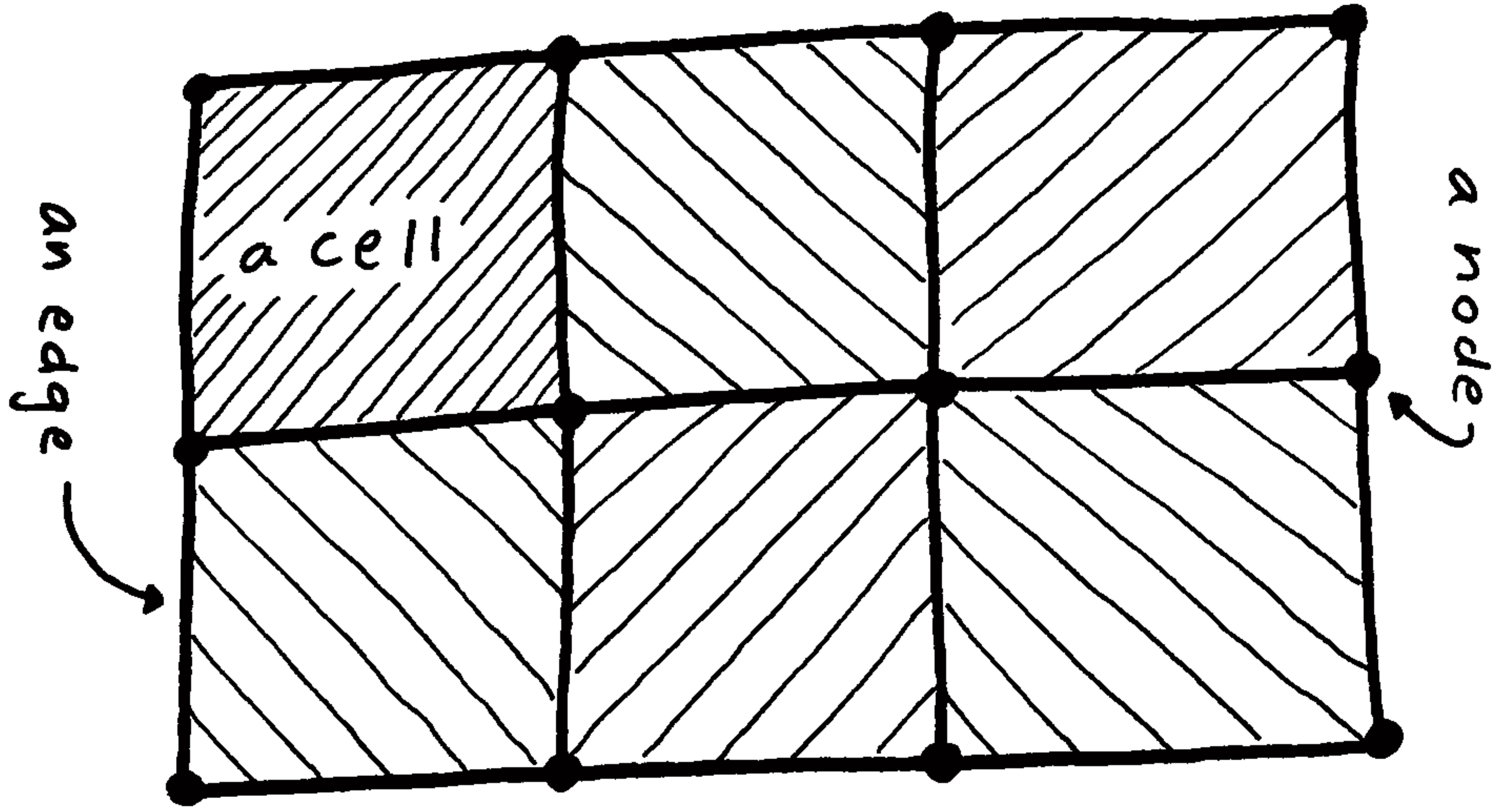


4.9

5

Cell Divisions

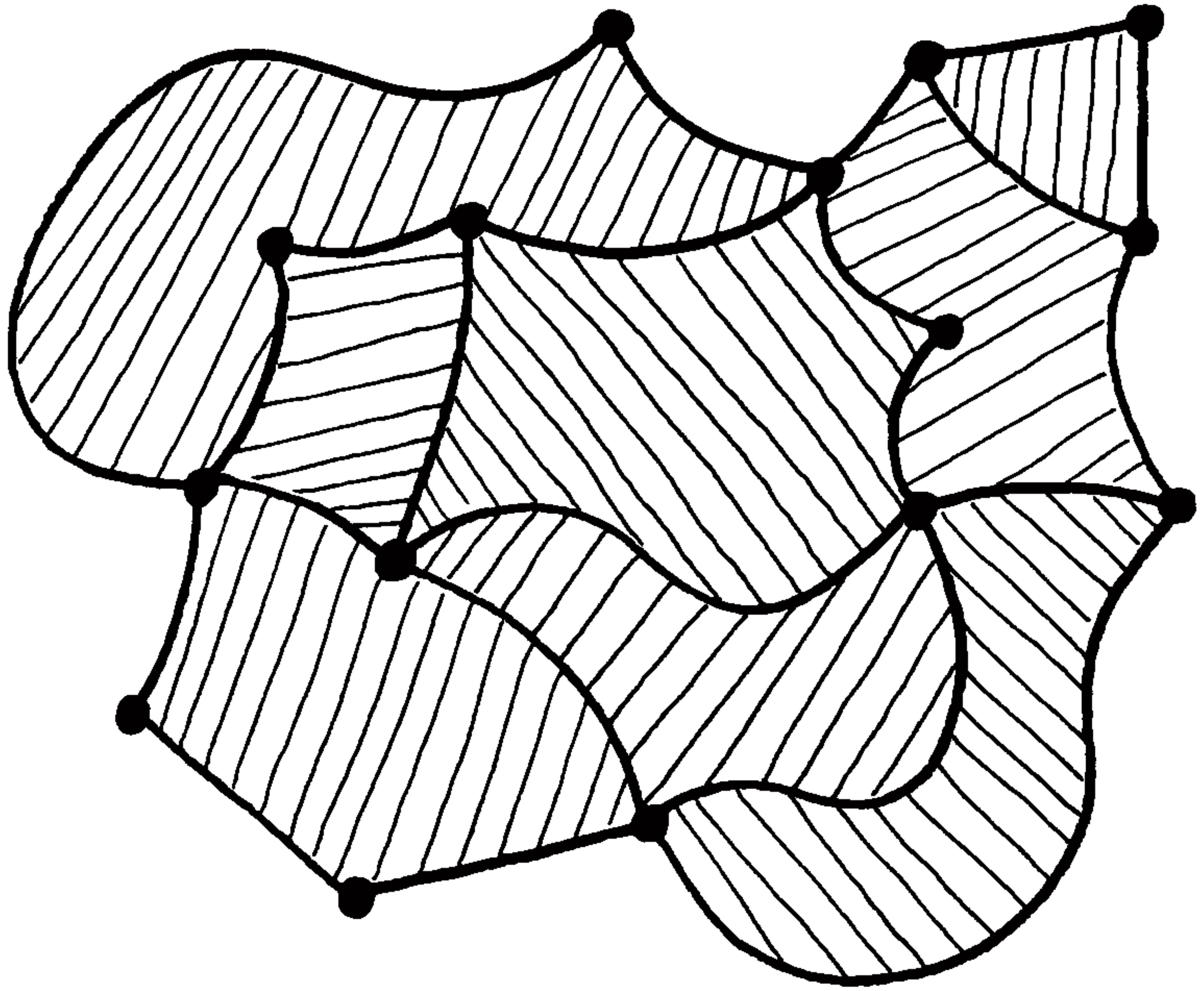
This is a cell division. It has nodes, edges, and cells.



The little squares are the cells. The whole thing fills a big rectangle.

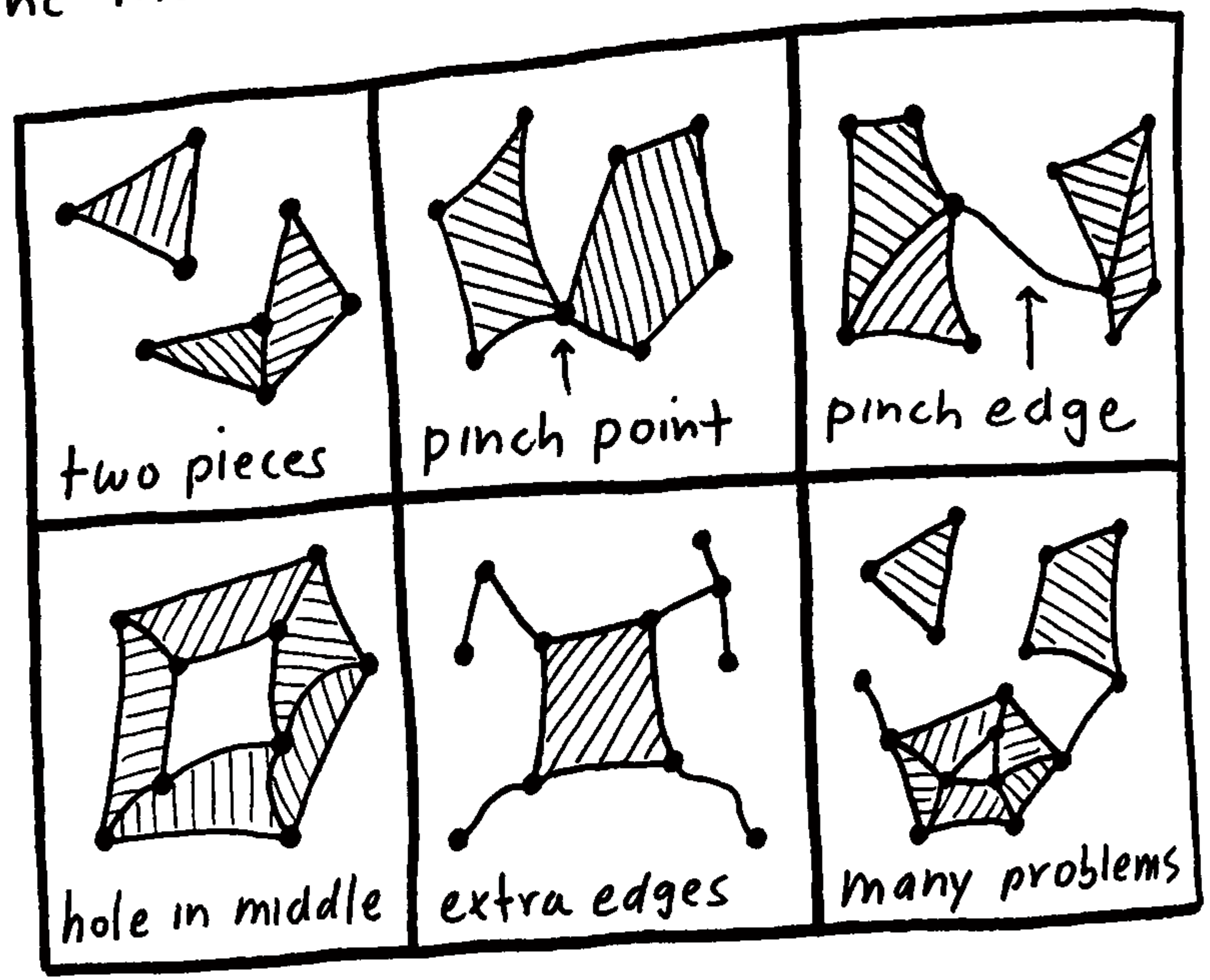
5.1

The cells don't have to be squares and the edges don't have to be straight...



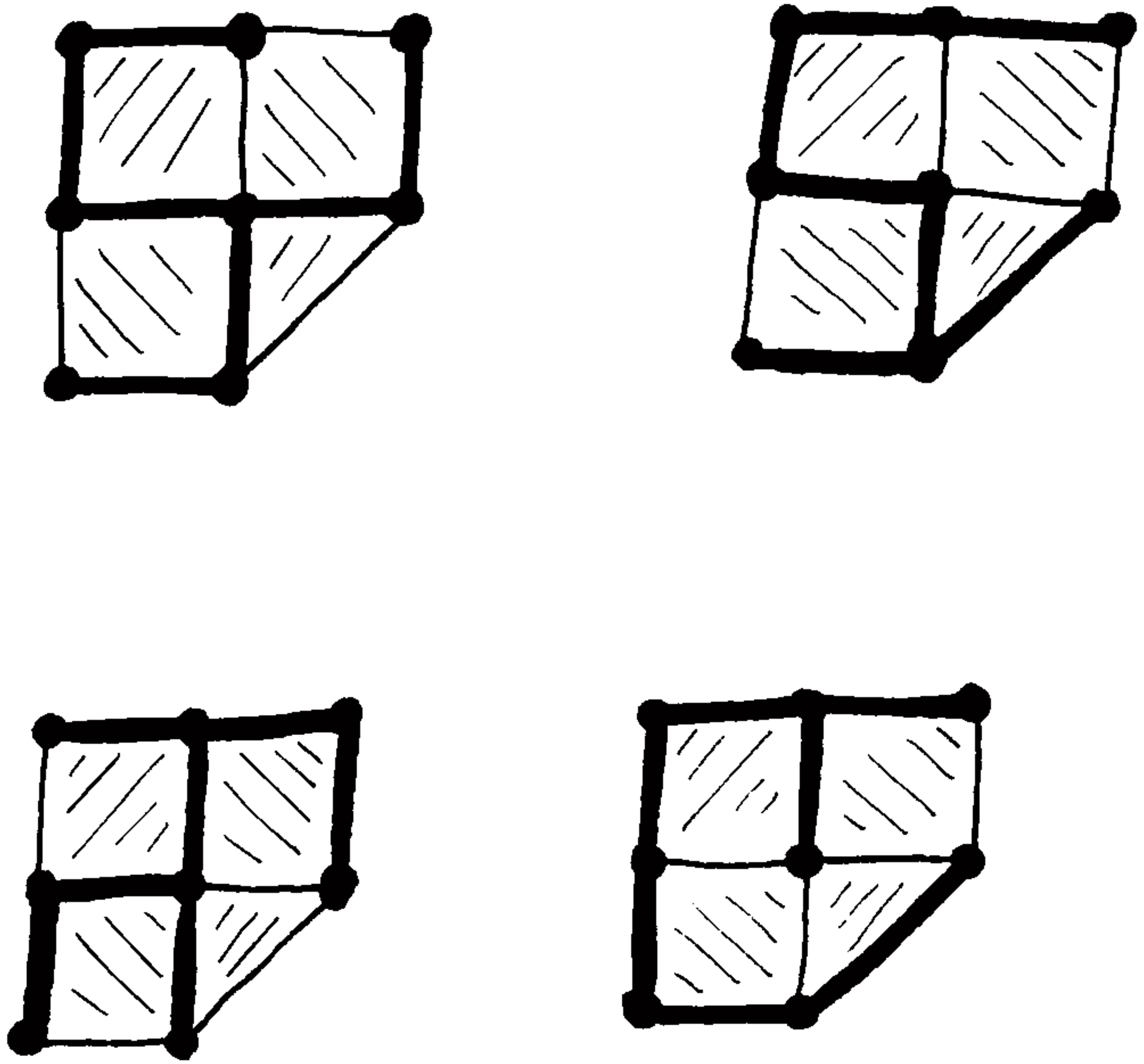
5.2

... but the whole thing has to fill the inside of one loop.



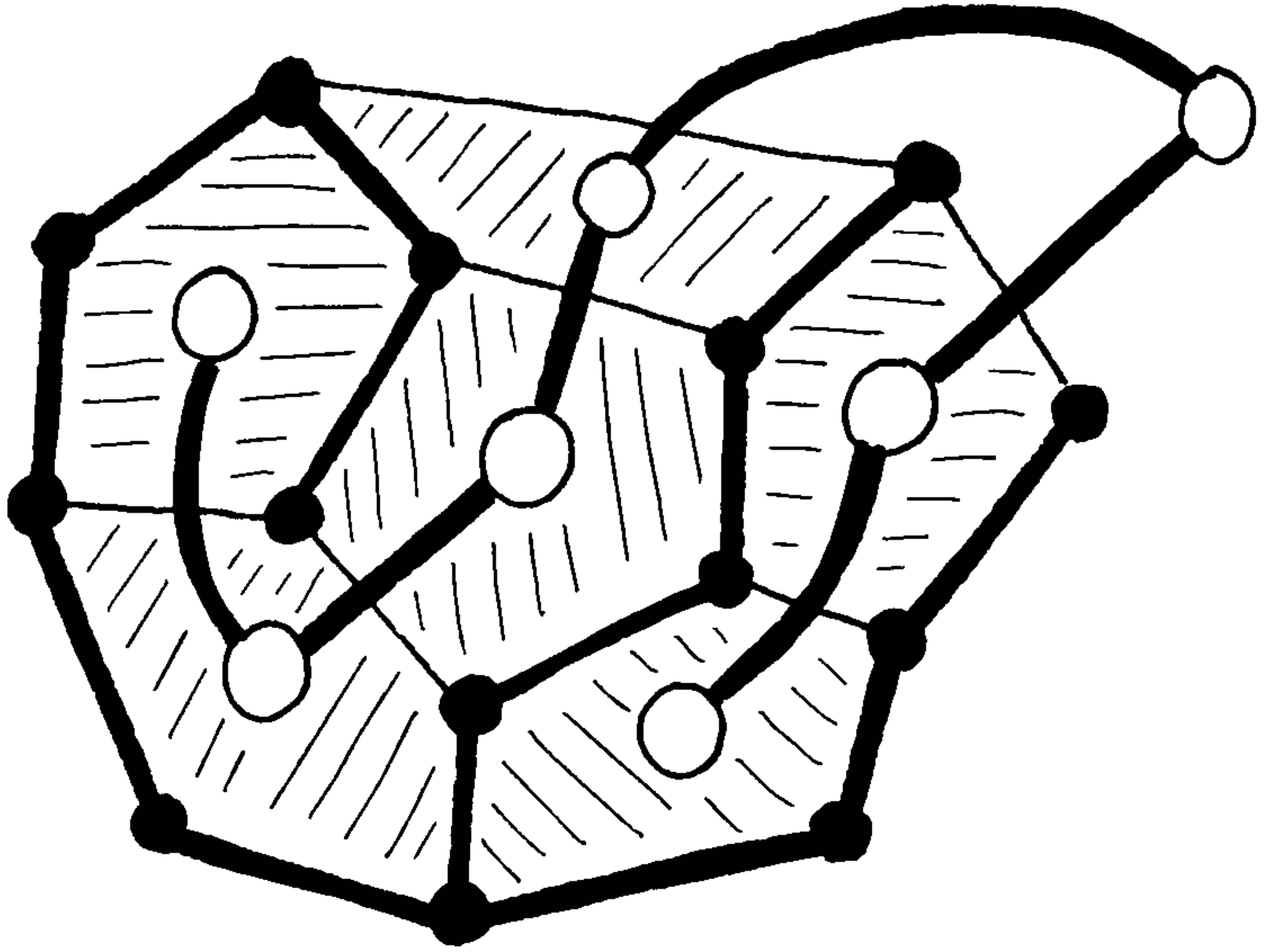
These things here don't count as cell divisions.

A cell division has many trees in it.



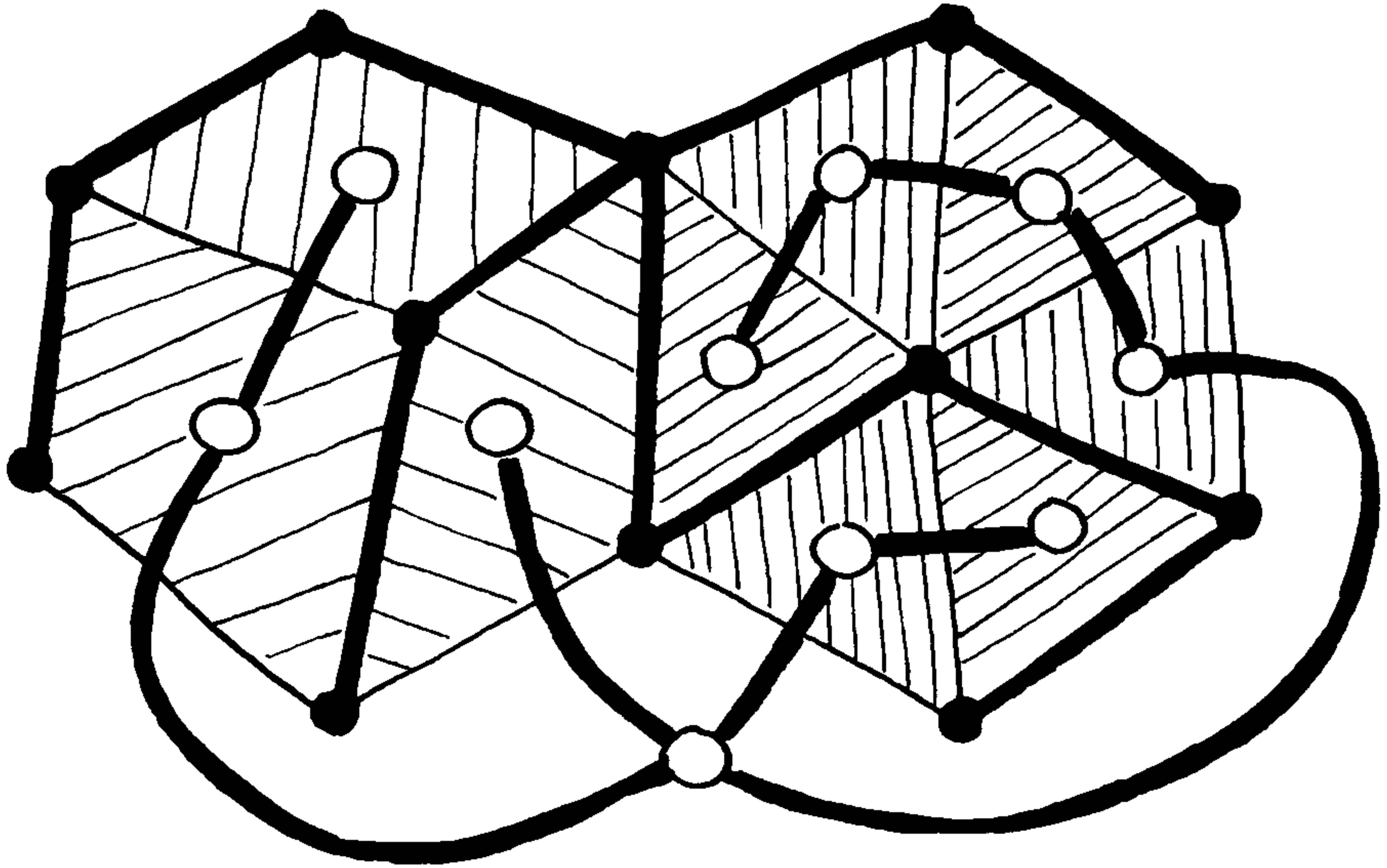
These are called spanning trees because they span all the nodes.

Each spanning tree has a partner tree that invades all the cells...



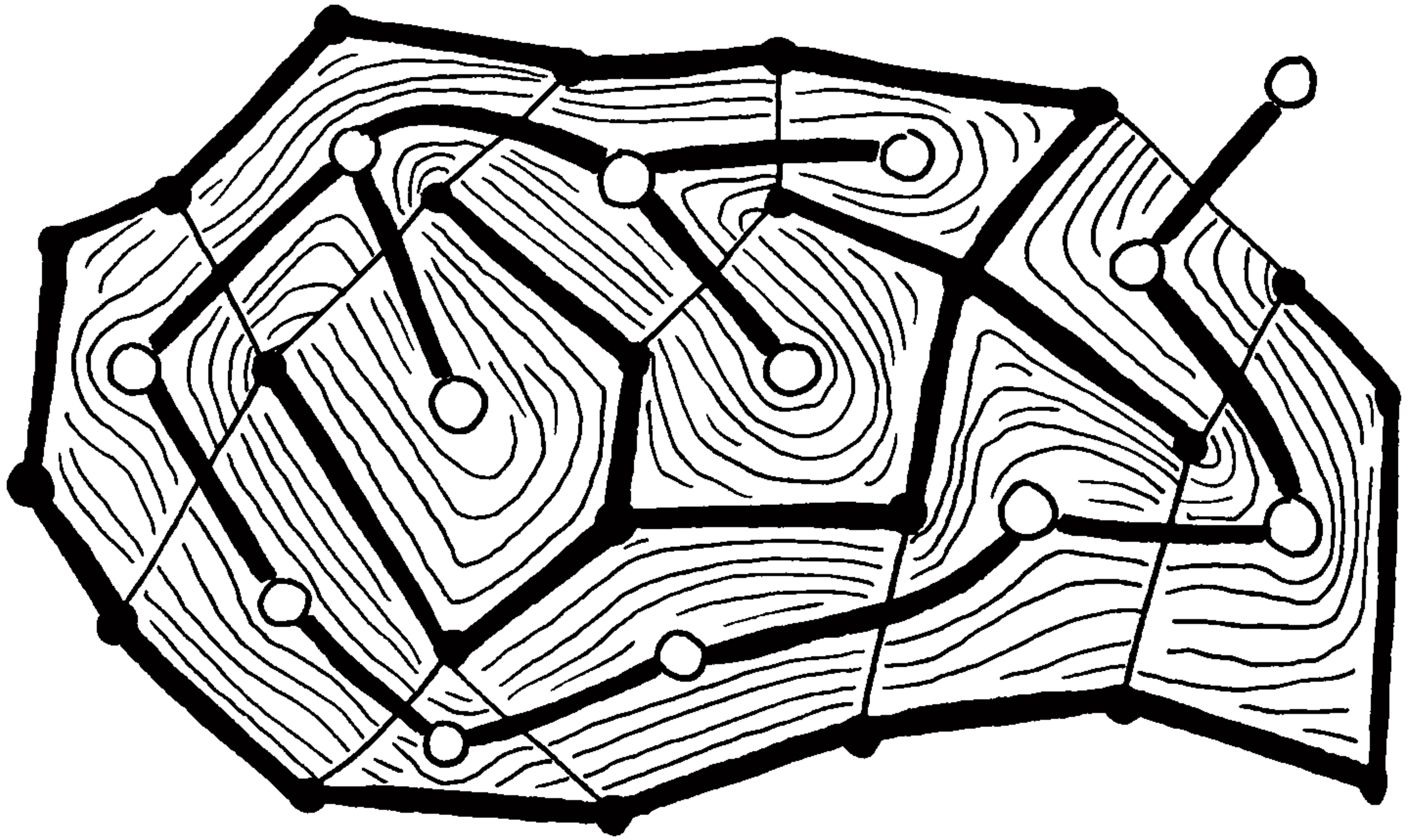
... crossing the edges unused by the spanning tree.

Every edge in the cell division is either in the spanning tree...

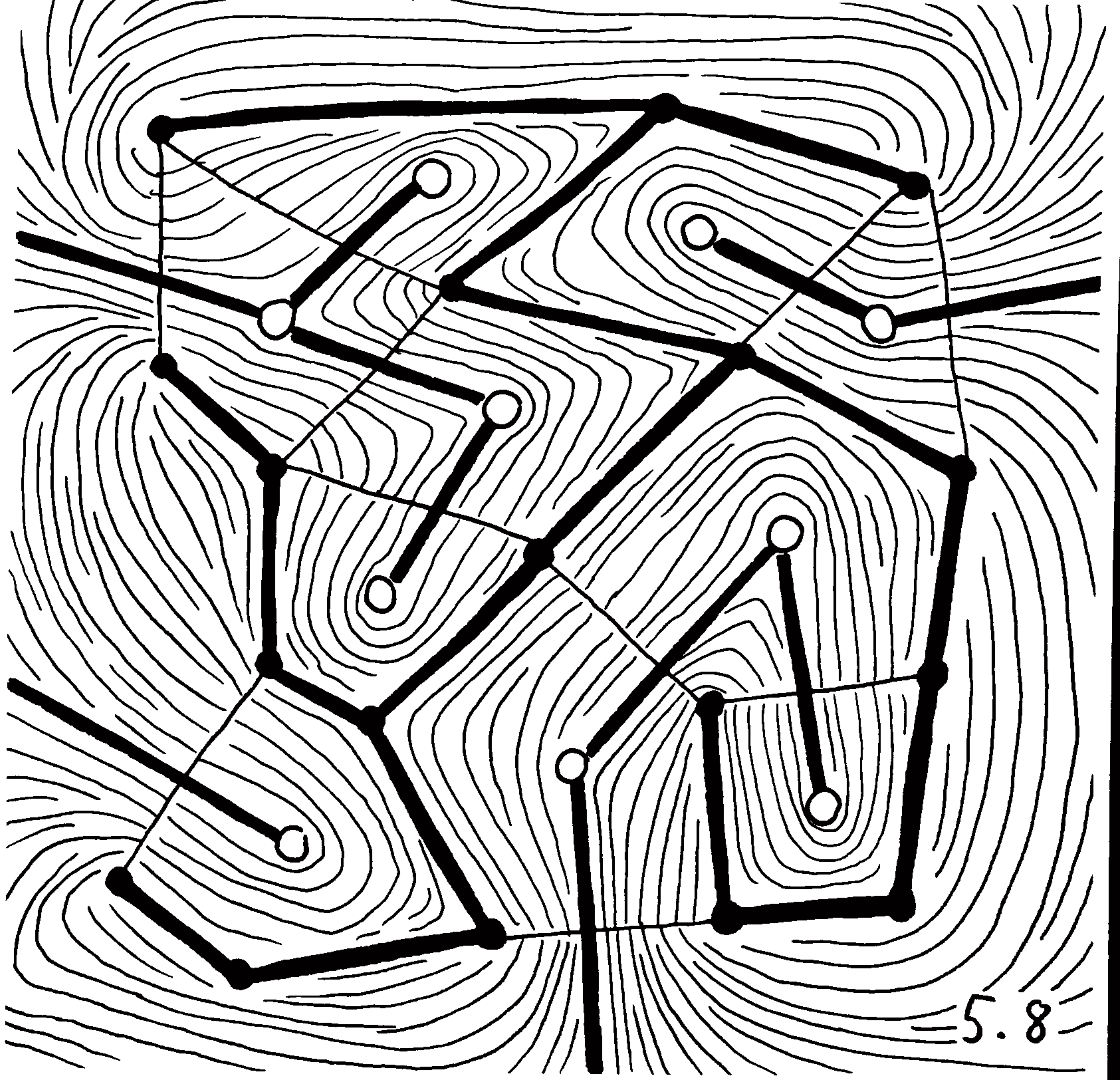


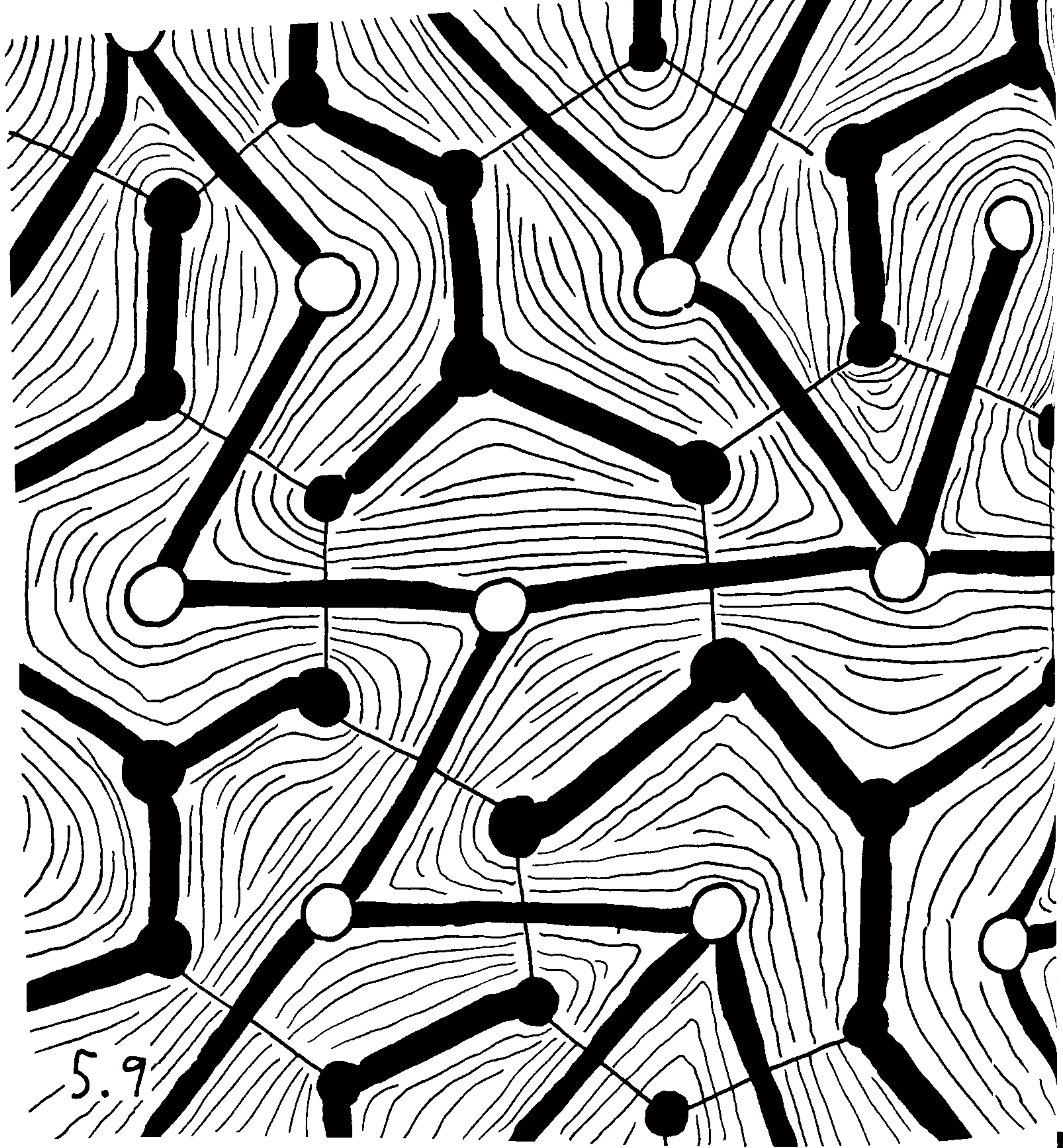
or crossed by an edge in the partner tree.

I like to think of a spanning tree and its partner as two banks with a river that flows between them.



Try drawing some of your own pictures like this. Play around, zoom in...

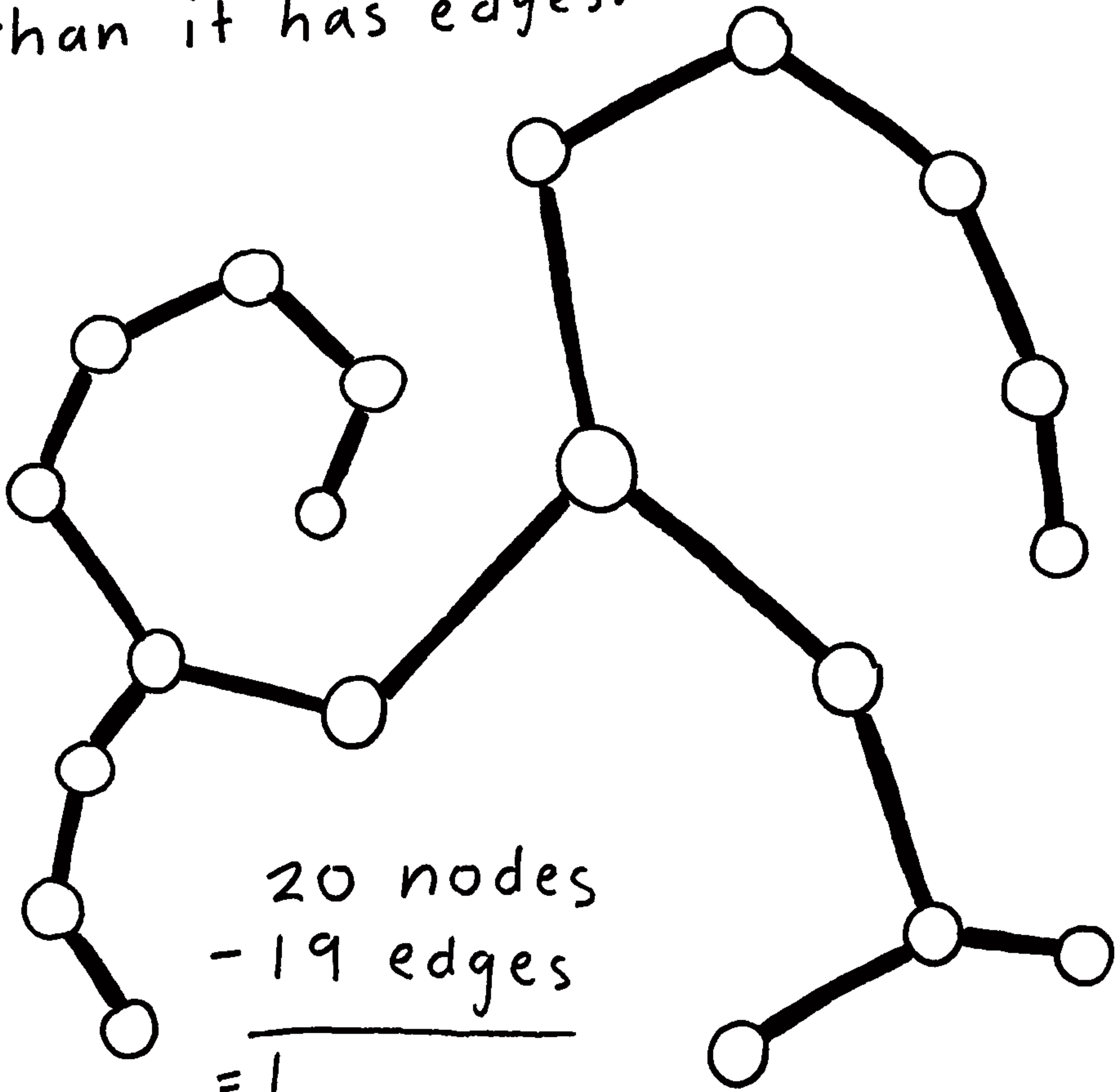




6

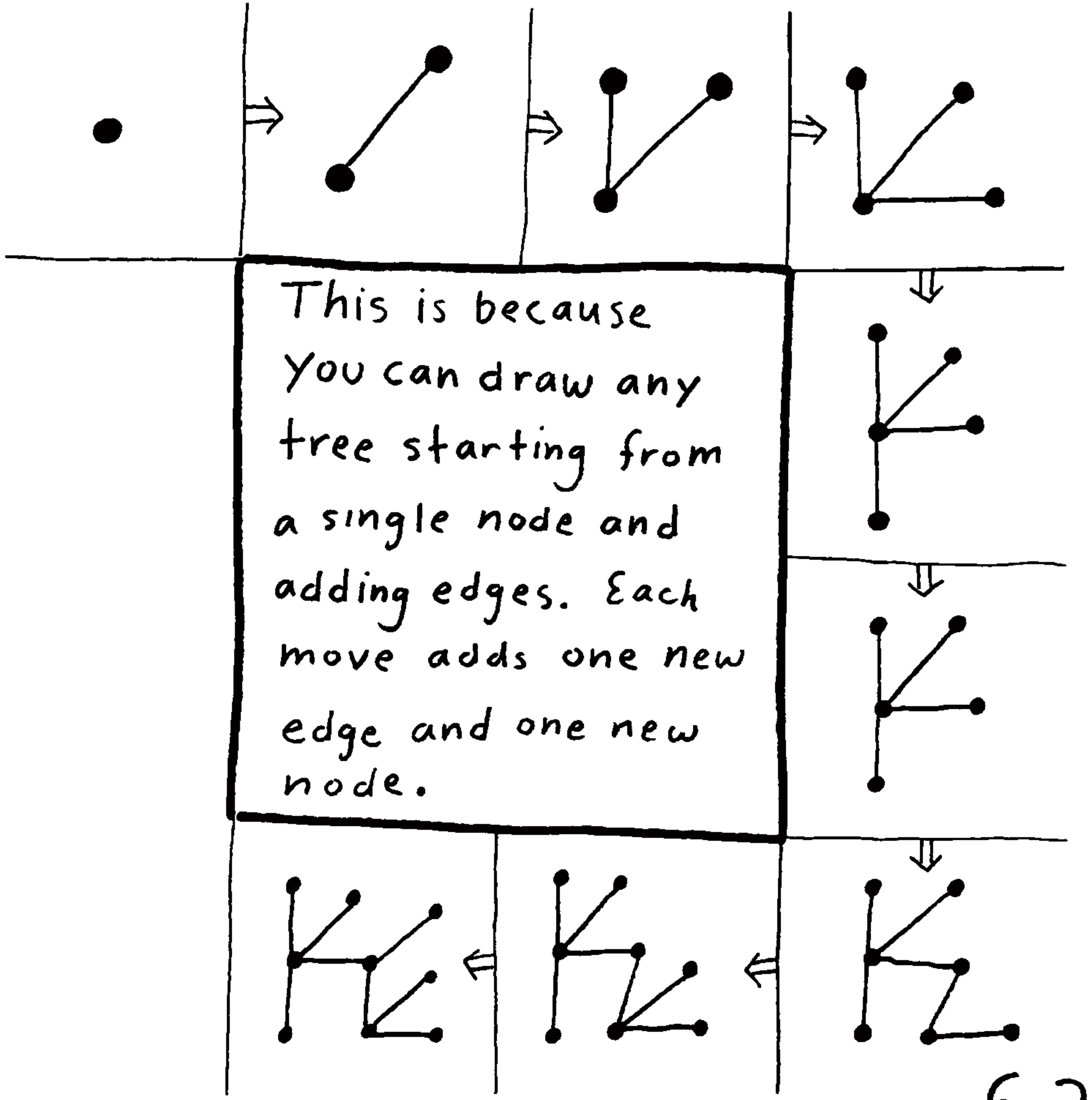
Euler's Formula

A tree has one more node than it has edges.



20 nodes
- 19 edges
= 1

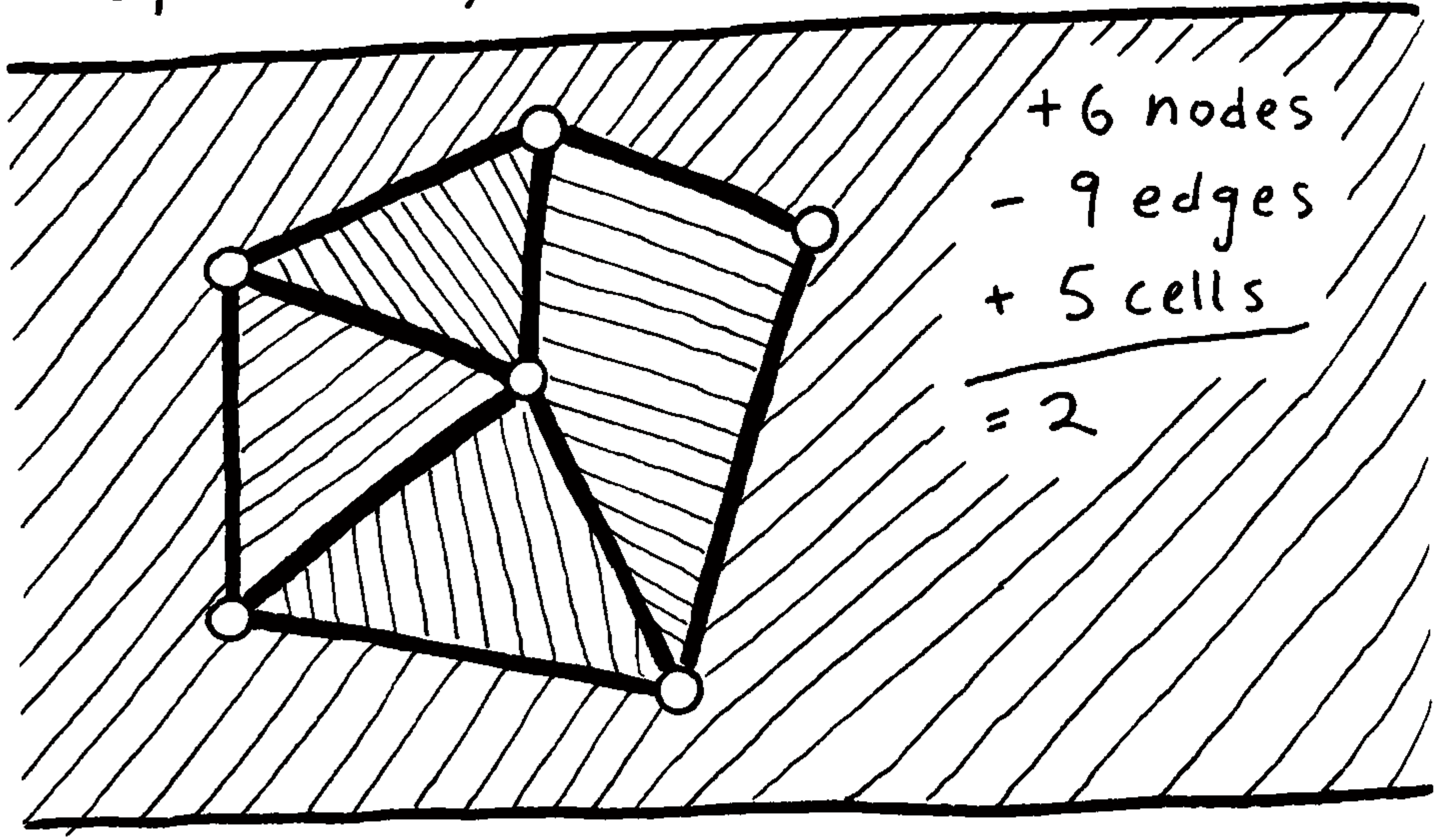
6.1



This is because you can draw any tree starting from a single node and adding edges. Each move adds one new edge and one new node.

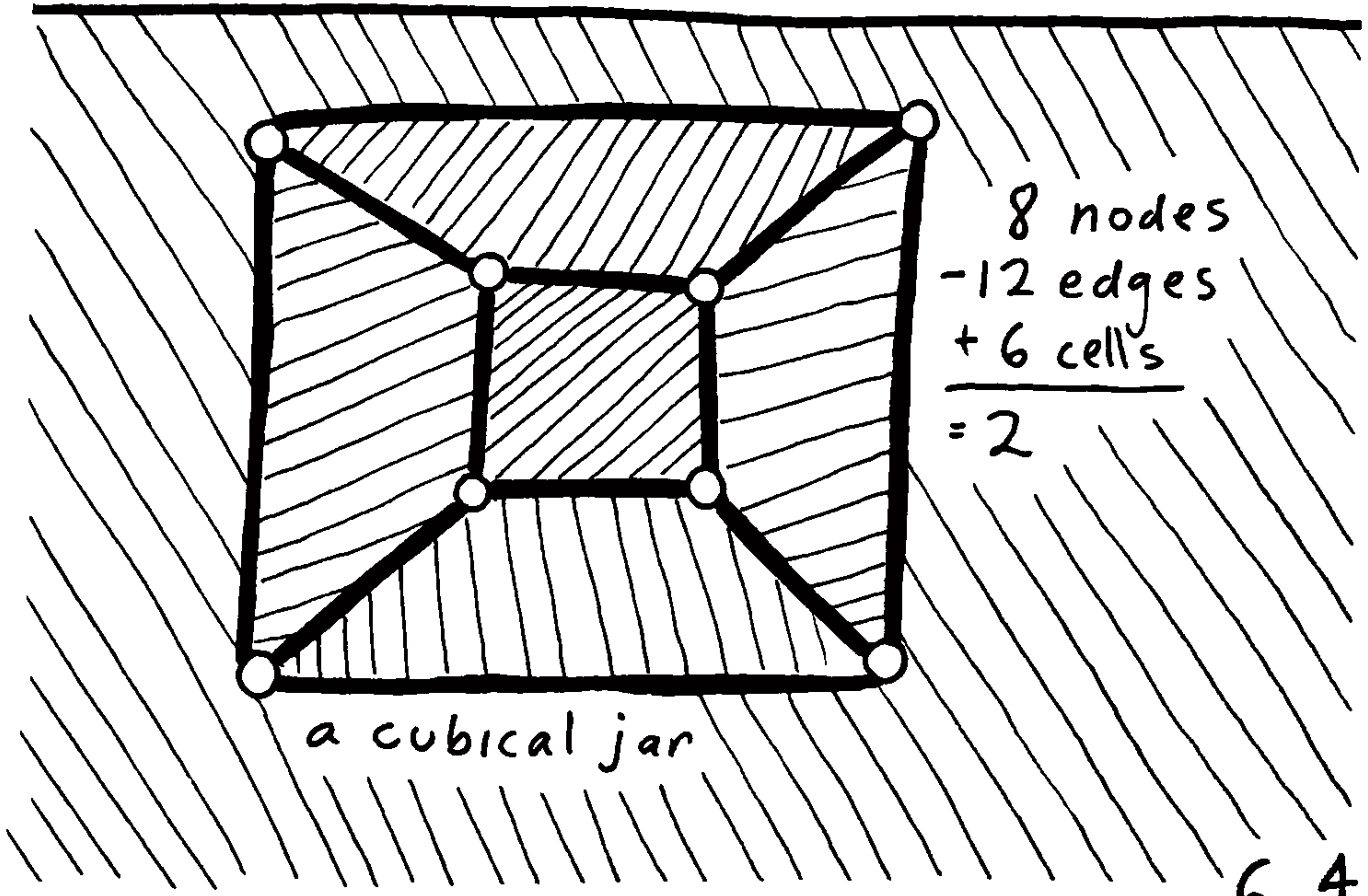
6.2

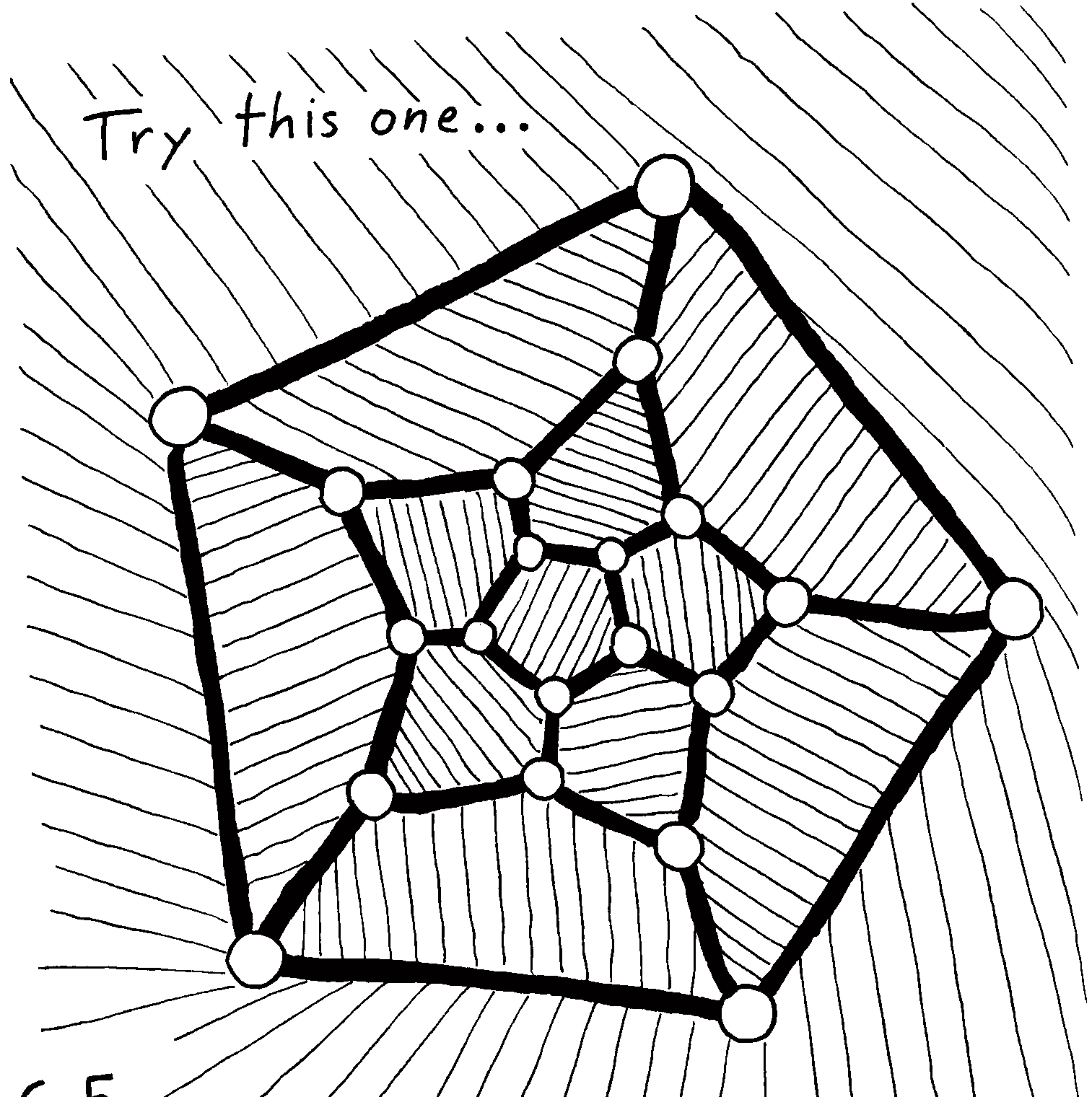
Count the parts of a cell division and total them up in a special way.

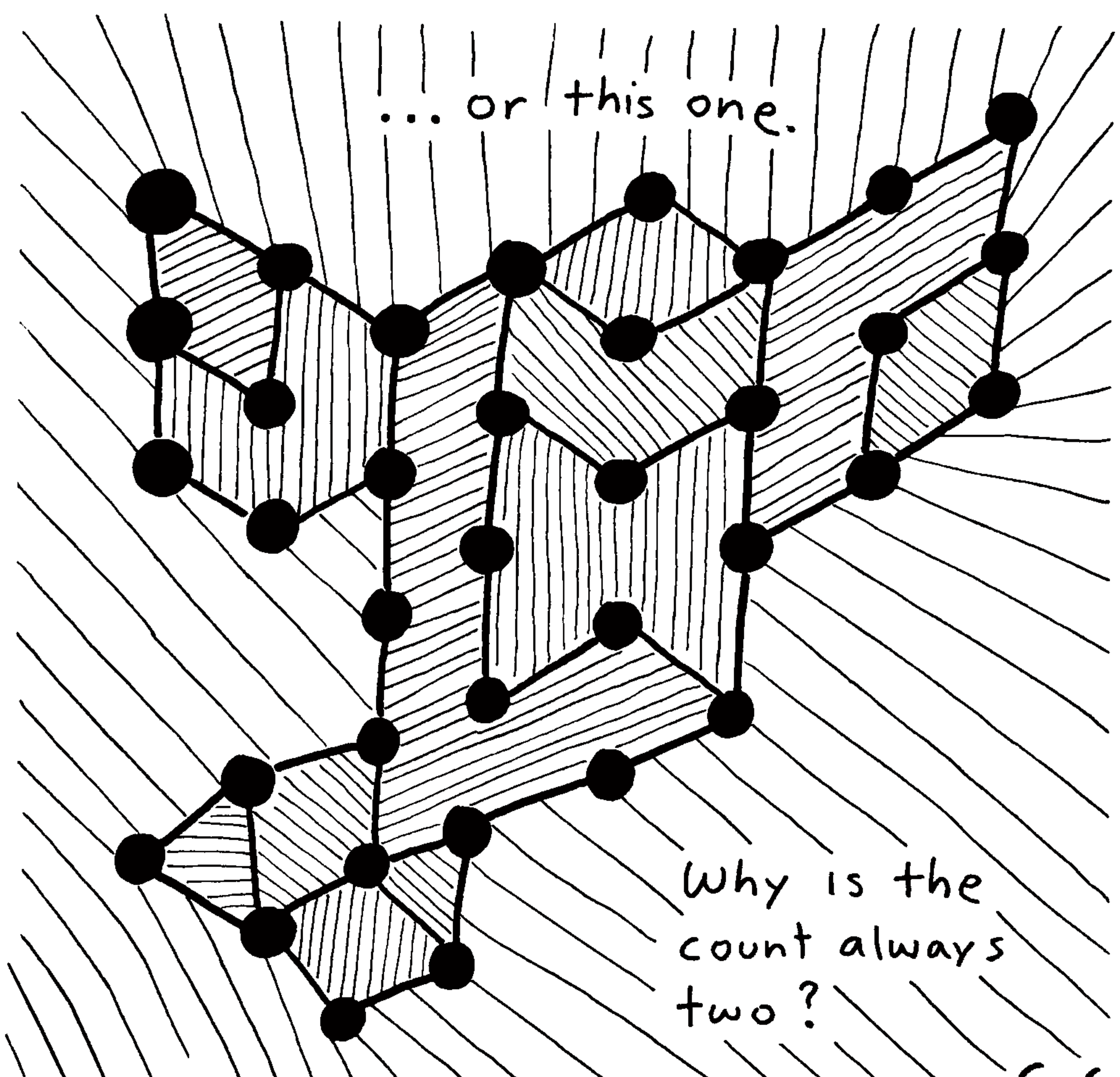


For this count, the outside also counts as a cell.

A cell division is like a jar without a lid. When you count the outside as a cell, it is like you are remembering the lid.

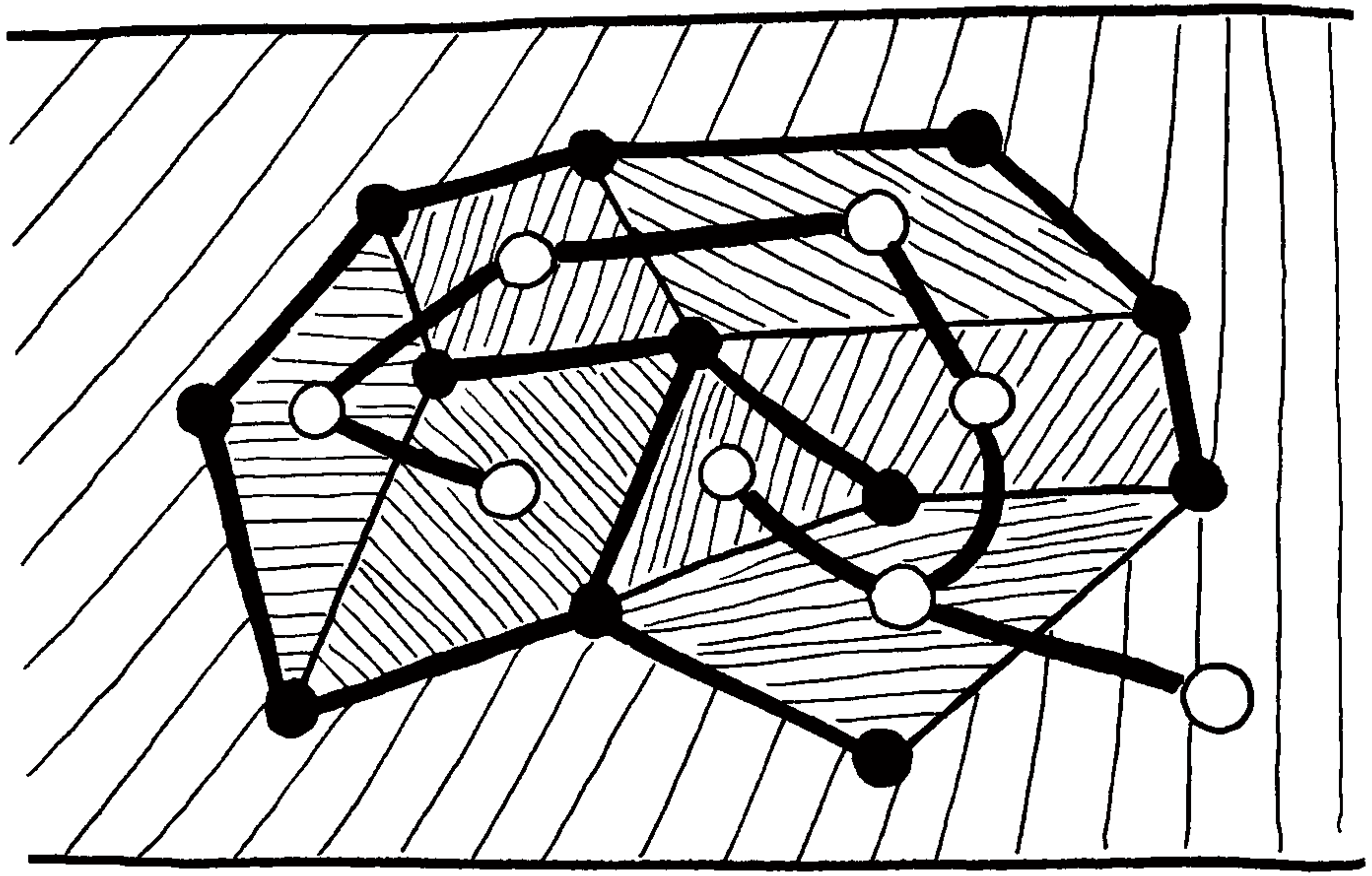



Try this one...


...or this one.
Why is the count always two?

One explanation:

Spanning and partner trees.



#cells = # partner tree nodes.

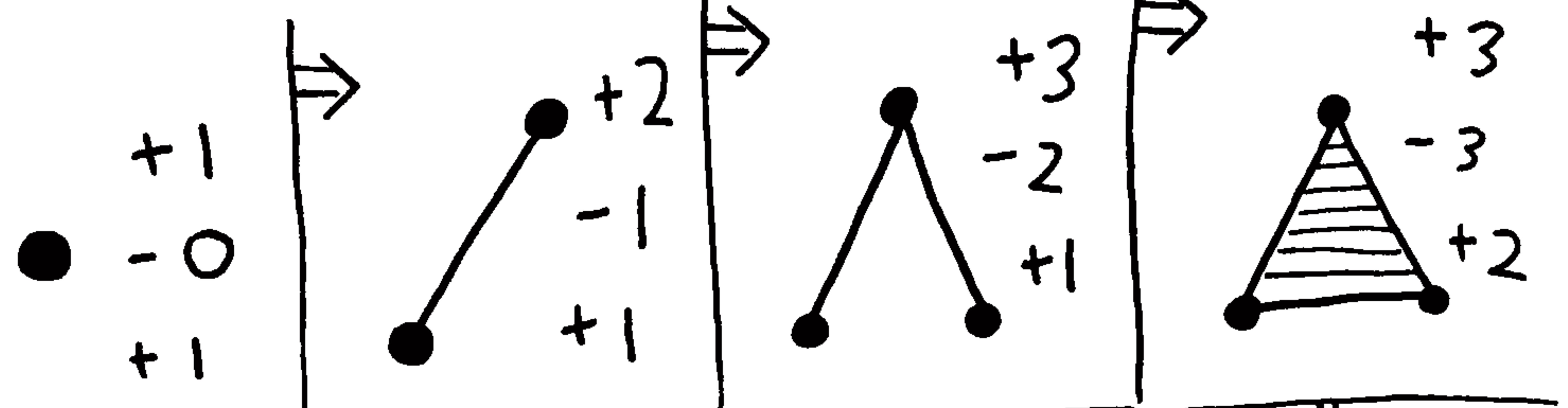
6.7

So... $\begin{bmatrix} \text{nodes} \\ -\text{edges} \\ +\text{cells} \end{bmatrix}$

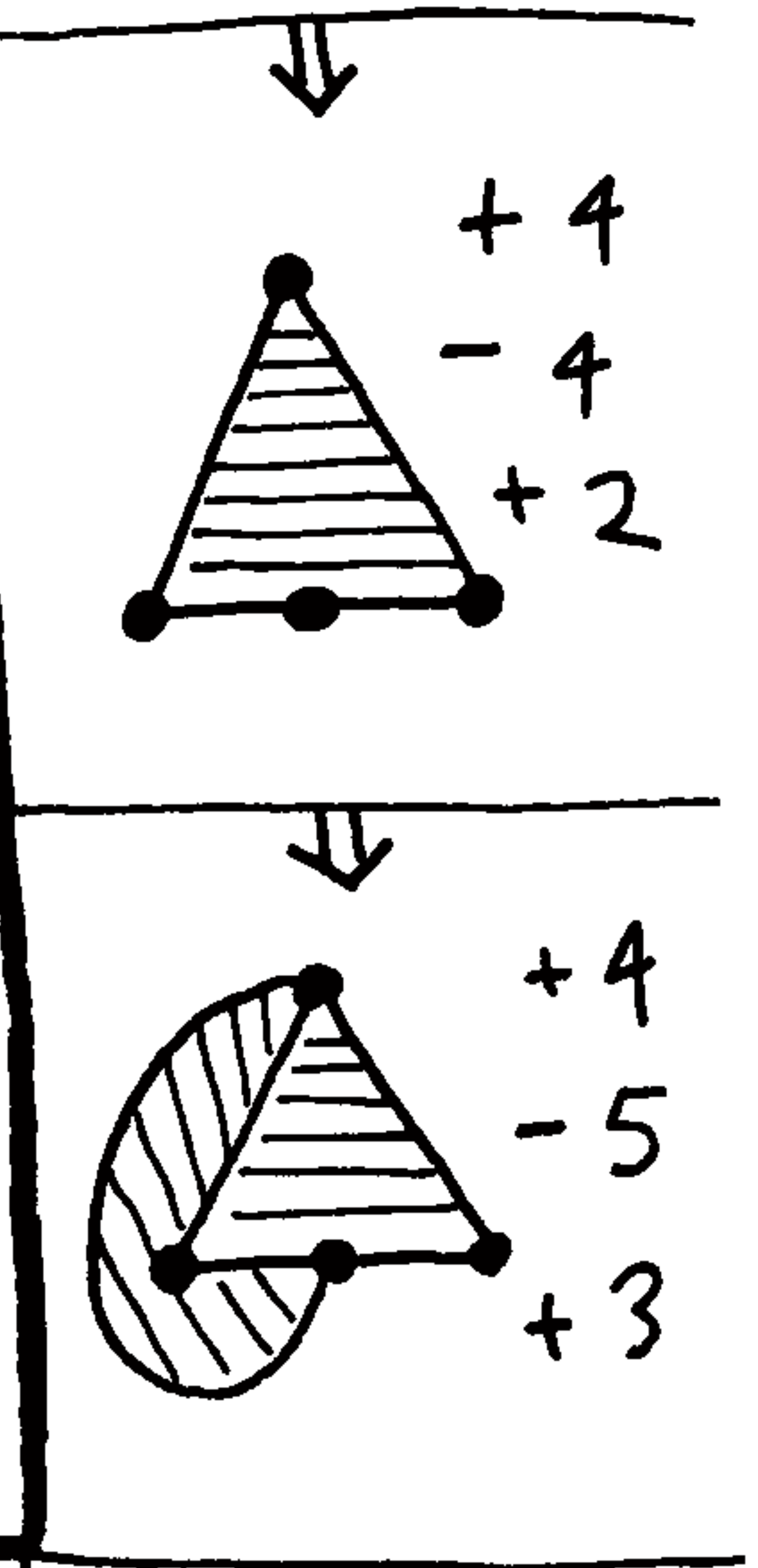
$= \begin{bmatrix} \text{spanning tree nodes} \\ - \\ \text{spanning tree edges} \end{bmatrix} + \begin{bmatrix} \text{partner tree nodes} \\ - \\ \text{partner tree edges} \end{bmatrix}$

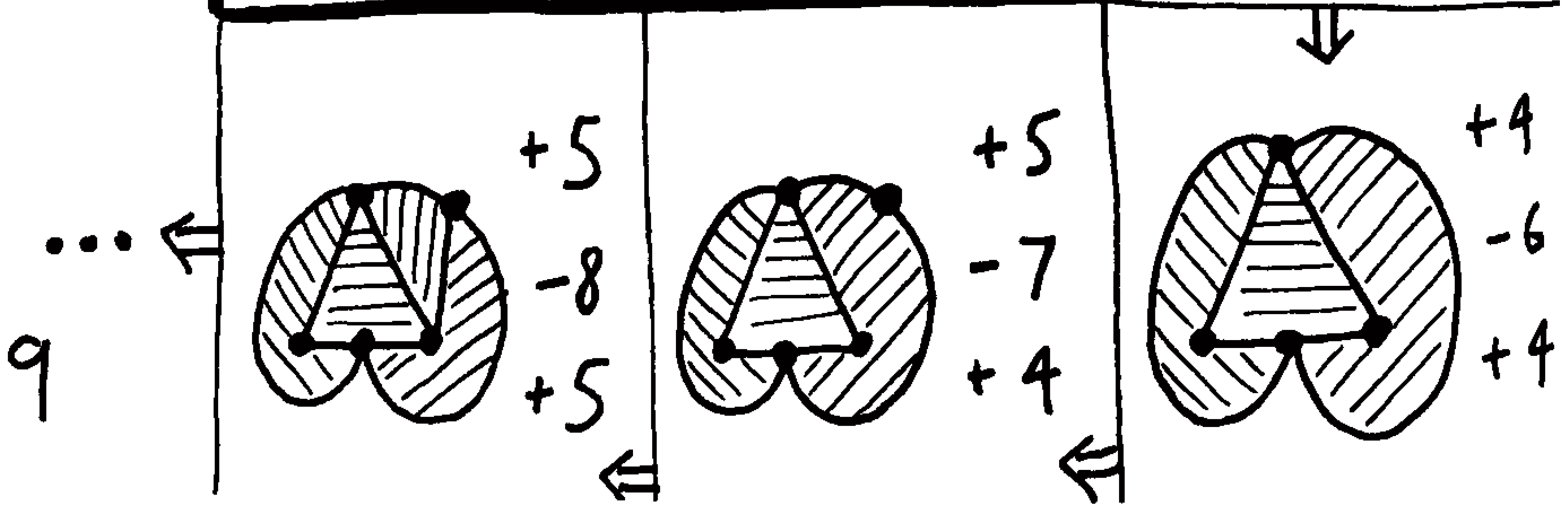
$= 1 + 1$

6.8



Another explanation: Build the cell division bit by bit and check that the count doesn't change.





7

Connecting Five Points