Cagli, Olson, Coxeter

Michele Emmer

In 1923 the mathematician Harold Scott MacDonald (Donald) Coxeter started his work on more than three-dimensional geometry. Coxeter writes in the preface to the book *Regular Polytopes*, published in 1948, [1] "the book grew out of an essay on Dimensional Analogy begun in February 1923. It is thus the fulfillment of 24 years' work", with an explicit reference to Edwin Abbott Abbott's book *Flatland: A Romance of Many Dimensions* [2] published without the author's name in 1884.

In chapter VII of his book *Ordinary Polytopes in Higher Space* Coxeter wrote: [3] "Polytope is the general terms of the sequence *point, segment, polygon, polyhedron*... A Polytope is a geometrical figure bounded by portions of lines, planes or hyperplanes: e. g. in two dimensions it is a polygon, in three a polyhedron. The word polytope seems to have been coined by Hoppe in 1882, and introduced into English by Mrs. Stott about twenty years later. Many simple properties of polytopes may be inferred by pure analogy: e. g. 2 points bound a segment, 4 segments bound a square, 6 squares a cube, 8 cubes a hyper-cube and so on." He added: "Only one or two people have ever attained the ability to visualize hyper-solids as simply and naturally as we ordinary mortals visualize solids; but a certain facility in that direction may be acquired by contemplating the analogy between one and two dimensions, then two and three, and so (by a kind of extrapolation) three and four." Coxeter recalls that when we try to understand the idea of a four-dimensional Euclidean space we are helped by imagining the efforts that a hypothetical two-dimensional being would make to visualize the three-dimensional world, exactly what happens in Flatland.

And pointed out that: "Little, if anything, is gained by representing the fourth Euclidean dimension as time. In fact, this idea, so attractively developed by H. G.

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M. Emmer, M. Abate (eds.), *Imagine Math 8*,
https://doi.org/10.1007/978-3-030-92690-8_7

Wells in *The Time Machine*, [4] has led such authors as J. W. Dunne (*An Experiment with Time* [5]) into a serious misconception of the theory of Relativity. Minkowski's geometry of space-time is *not Euclidean*, and consequently has no connection with the present investigation."

An even more effective way, again based on analogy, was suggested by Poincaré in 1891 [6]:

"In the same way that we draw the perspective of a three-dimensional figure on a plane, so we can draw that of a four-dimensional figure on a canvas of three (or two) dimensions. To a geometer this is but child's play. We can even draw several perspectives of the same figure from several different points of view. We can easily represent to ourselves these perspectives, since they are of only three dimensions. Imagine that the different perspectives of one and the same object occur in succession . . . There is nothing, then, to prevent us from imagining that these operations are combined according to any law we choose—for instance, by forming a group with the same structure as that of the movements of an invariable four-dimensional solid. In this there is nothing that we cannot represent to ourselves, and, moreover, these sensations are those which a being would experience who has a retina of two dimensions, and who may be displaced in space of four dimensions. In this sense we may say that we can represent to ourselves the fourth dimension."

The one who first studied and determined the six regular solids of four-dimensional space was Ludwig Schläfli. Using the Coxeter nomenclature they are regular simplex {3, 3, 3}, hypercube {4, 3, 3}, 16-cell {3, 3, 4}, 24-cell {3, 4, 3}, 120- cell {5, 3, 3}, 600-cell {3, 3, 5}. Schläfli's work was not at all appreciated and almost all of his works were not accepted for publication. Only six years after his death, in 1901, the *Theorie der vielfachen Kontinuität* was published, [7] in which Schläfli dealt with n-dimensional geometry and in particular with four-dimensional solids (which he called Polyschem). Some excerpts from this work was published in English and French in 1855 and 1858 but went completely unnoticed, probably due to the fact that, as Coxeter observed, "their dry-sounding titles tended to hide the geometrical treasures that they contain, like the art of van Gogh" [8].

Coxeter wrote: [9] "The discoverers and earlier rediscoverers of the regular polytopes (Schläfli, Stringham, Forchhammer, Rudel and Hoppe) all observed that the total number of even-dimensional elements and the total number of odd-dimensional elements are either equal (as in the case of polygon) or differ by 2 (as in the case of convex polyhedron)".

Many looked for a general formula valid in every dimension. It was Poincaré in 1893 who wrote on the subject a short note which he expanded six years later. In 1893 Poincarè published the first work dedicated to Topology (or analysis situs) and Coxeter recalls that we are exactly in the field of topology with these types of results [9]:

"It must be emphasized that the theorem 9-11is a theorem of topology, which is more general than the ordinary geometry in that it is not concerned with measurement, nor even straightness." Theorem 9–11 is the proof of Euler's formula for Polytopes in all dimensions. (9 is the chapter of Coxeter's book, 1 indicates the paragraph). The lack of attention to Schläfli's works was the reason why

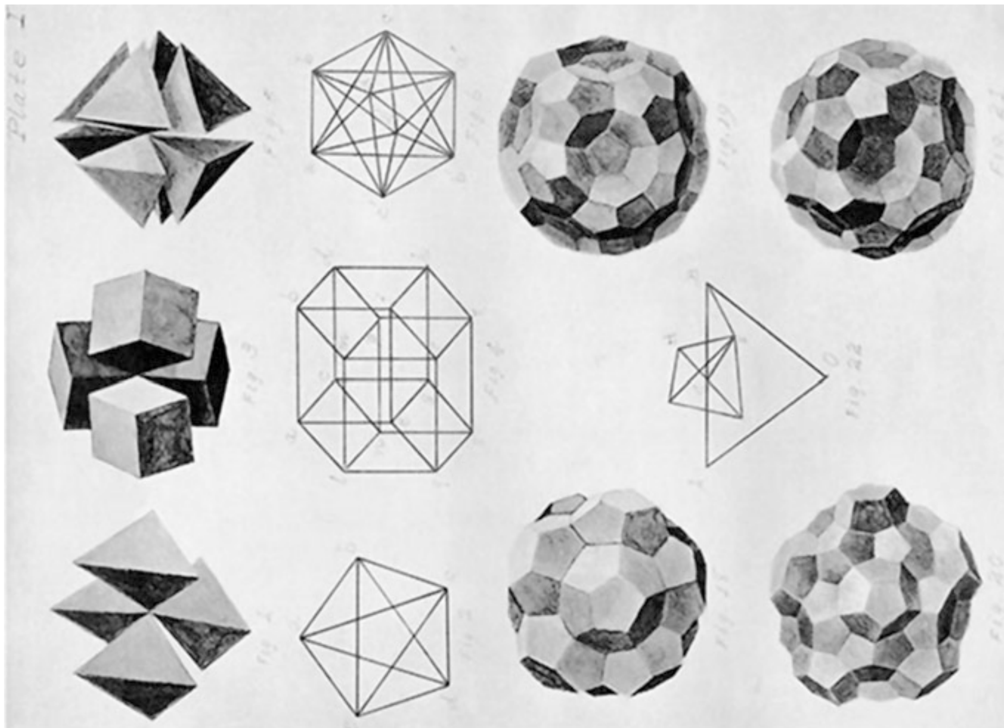

Fig. 1 W. I. Stringham, Regular Figures in n-Dimensional Space, 1880 [10]

many believe that Washington Irving Stringham was the first to determine the regular figures of four-dimensional space in his article *Regular Figures in n-Dimensional Space*, [10] published nearly thirty years after Schläfli's work. The figure of the hypersolids in four dimensions created by Stringham identifies the three-dimensional elements that make up the hypersolids and approaches them in a more or less random way. It was difficult to have a precise idea of their structure, however Stringham's work achieved indisputable success (Fig. 1).

It is no coincidence that the art historian Linda D. Henderson, in her extensive essay dedicated to the influences on art of ideas inspired by the fourth dimension and non-Euclidean geometry, despite having also thoroughly studied the mathematical aspects of the issues dealt with, as shown by extensive specialized bibliography, do not cite Schläfli's work. Speaking of Stringham, Henderson notes that the impact of the article was remarkable, so much so that there are numerous references to it in the writings of mathematicians and non-mathematicians of the early twentieth century [11]. Between 1900 and 1910 the different notions on the fourth dimension, developed in the previous century, spread more and more, even outside the scholars' circle. This phenomenon became more widespread in the United States, where a large number of popular magazines provided ample space to discuss the novelty, and in Russia. Interest peaked in 1909, when *Scientific American* sponsored "the best explanation of four-dimensional geometry", receiving 245 contributions from around the world. As Henderson pointed out, the fourth dimension was interpreted by all participants as a purely spatial phenomenon; time was never mentioned as a fourth dimension.

Linda Henderson makes it clear that, in the literature on the fourth dimension in the late nineteenth and early twentieth centuries, between the two possible interpretations of the fourth dimension, time was always the least important. In a more philosophical and mystical view of the fourth dimension, the role of time was to visualize a higher dimensional space, but time itself was not interpreted as a fourth dimension. Rather, it was the geometry of higher-dimensional spaces, along with non-Euclidean geometries, that fascinated the public in the early twentieth century.

An important role in the popularization of the fourth dimension was played by Abbott's volume, which was immediately a great success; a second edition was published in 1884 and it had nine reprints up to 1915. Mathematicians and writers alike cited *Flatland* on several occasions. Among others, Charles Howard Hinton, a great enthusiast of the philosophy of the fourth dimension, who published several books dedicated to it between 1880 and 1904. In 1907, he published *An Episode of Flatland*, [12] a sort of reworking of Abbott's novella: this was not the only attempt at such rewriting, although no imitator has achieved Abbott's inventiveness or humour.

There was already some confusion between four-dimensional Euclidean space and space-time with time as fourth dimension in Abbott's book, even if the theory of relativity obviously did not exist in 1884. Surely, the encounter between the Sphere and the Square in *Flatland* will contribute a lot to the misunderstanding: the encounter is described by Arthur Eddington in the book *Space, Time and Gravitation* (1920) [13], a classic in the popularization of the theory of relativity, as the best popular exposition of the fourth dimension. Eddington was thinking about the four-dimensional space of space-time, and wondered to what extent the world imagined by Abbott agreed with the space-time of relativity. There are three points in the narrative that Eddington was highlighting. First of all, the fact that when a four-dimensional body moves, its three-dimensional section can vary; in this way it is possible for a rigid body to alter its shape and dimensions. Moreover, a four-dimensional body can enter a completely enclosed three-dimensional room, just as a three-dimensional being can place a pencil anywhere within a two-dimensional square without intersecting its sides. This is how the Sphere behaves when it visits Flatland; the Square naturally fails to see the visitor. Finally, it becomes possible to see the inside of a solid in three dimensions just as a three-dimensional being can see the inside of a square by looking at it from a point outside the plane on which it lies.

The mathematician Jouffret wrote the two volumes *Traité élémentaire de géométrie à quatre dimensions* in 1903 [14] and *Mélanges de géométrie à quatre dimensions* in 1906 [15]. The method used by Jouffret to visualize objects in four dimensions was a kind of descriptive geometry in which these objects were projected onto the two-dimensional plane, i.e. the paper on which they were drawn. In many cases, the objects were rotated in order to obtain additional images that gave more information about their dimensions.

Of course, it should be immediately stated that in no way is a direct cause-and-effect relationship suggested between *n-dimensional* geometry and the development of the art of Picasso and Braque. The main sources of Cubism are to be found in art itself, first of all, in African art and in the paintings by Cézanne.

There is another mathematician who played a role in the development of some Cubists, and particularly in that of Cubism theorists Gleizes and Metzinger. It is Metzinger himself who explains the role of Maurice Princet, who worked in an insurance company. He conceived mathematics as an artist might, and he evoked *n-dimensional* space as a scholar of aesthetics would. He wanted to push painters towards the new ideas about space opened up by Victor Schlegel. Schlegel was one of the mathematicians who had contributed most to the emergence of *n-dimensional* geometry at the end of the nineteenth century, and who had produced three-dimensional models of hypersolids in four dimensions. Coxeter mentions him in his book: [1] "The theory of regular honeycombs in hyperbolic space but I have resisted the temptation to add a fifteenth chapter on that subject."

In the final version of *Les peintres cubistes* Apollinaire writes [16]:

"Les nouveaux peintres ne se sont proposé d'être des géomètres. Mais on peut dire que la géométrie est aux arts plastiques ce que la grammaire est à l'art de l'écrivain. Or, aujourd'hui, les savants ne s'en tiennent plus aux trois dimensions de la géométrie euclidienne. Les peintres ont été amenés tout naturellement et, pour ainsi dire, par intuition, à se préoccuper de nouvelles mesures possibles de l'étendue que dans le langage des ateliers modernes on désignait toutes ensemble et brièvement par le terme de *quatrième dimension*... Elle est l'espace même, la dimension de l'infini ; c'est elle qui doue de plasticité les objets." ("The new painters do not claim to be scholars of geometry. But it is safe to say that geometry is to visual arts as grammar is to the art of writing. Nowadays, scholars are no longer limited to Euclid's three dimensions. Painters have very naturally, one might say instinctively, explored the new possibilities of space which, in the language of modern art, are referred to as the fourth dimension. The fourth dimension is space itself, the dimension of infinity; the fourth dimension gives objects plasticity.")"

Henderson remarks that, apart from specific applications, the fourth dimension played an important role in the development of an idealism suited to Cubist philosophy.

Umberto Boccioni in 1913 discussed in detail the role of the fourth dimension in Futurist art. In 1914 he collected his observations in *Pittura scultura futuriste (Dinamismo Plastico)* (Futurist painting and sculpture: plastic dynamism): [17] "Dynamism is a lyrical conception of shapes interpreted as part of the infinite manifestation of the relationship between their absolute and relative motion, between environment and object, until they form the appearance of a whole: *environment +object*... Between rotation and revolution, in short, is life itself, captured in the shape that life creates in its *infinite succession*... We come to this succession... through an intuitive search for a unique shape that gives continuity in space... to dynamic continuity as a unique shape. And it is not by chance that I say shape and not line, because dynamic shape is a kind of fourth dimension in painting and sculpture, which cannot live perfectly without the complete affirmation of the three dimensions that determine volume."

Boccioni recalls that the Cubists claimed to fully understand the idea of the fourth dimension:

"I remember having read that Cubism with his breaking up of the object and unfolding of the parts of the object on the flat surface of the picture approached the fourth dimension . . . If with artistic intuition it is ever possible to approach the concept of the fourth dimension, it is we Futurists who are getting there first. In fact, with the unique form that gives continuity in space we create a form that is the sum of the potential unfolding of the three kwon dimensions. Therefore, we cannot make a measured and finite fourth dimension, but rather a continuous projection of forces and forms intuited in their infinite unfolding."

Boccioni was interested in the passage of a higher-dimensional form through our three-dimensional space, in obtaining a continuous shape via this passage, as in his famous 1913 sculpture *Unique Forms in the Continuity of Space* (Fig. 2).

In 1917, in *La peinture d'avant-garde* [18] Gino Severini clarified how the links between Futurist art and geometry were to be understood, and looked at Poincaré: [19] "*L'espace ordinaire se base en général sur la convention inamovible des 3 dimensions; les peintres, dont les aspirations sont illimitées, ont toujours trouvé trop étroite cette convention. C'est-à-dire qu'aux 3 dimensions ordinaires, ils tâchent d'ajouter une 4e dimension qui les résume et qui est différemment exprimée, mais que constitue le but de l'art des toutes les époques. Boccioni, en définissant ce qu'il appelle le "dynamisme", fait allusion à une sorte de 4e dimension, qui serait "la forme unique donnant continuité dans l'espace", . . . Il s'agit de trouver une définition le plus possible simple et vraie, au point de vue artistique. C'est pourquoi, j'ai cherché dans la géométrie qualitative (*Analysis Situs de Poincaré*) la démonstration plus évident de cette 4e dimension, en sachant d'avance, que la science géométrique ne pourrait que soutenir des convention déjà établies par l'intuition artistique de nous tous. Si j'aime chercher souvent un appui sur les vérités de la science, c'est que je vois là un excellent moyen de contrôle et d'ailleurs aucun de nous saurait négliger les notions que la science met à notre portée pour intensifier notre sens du réel.*"

Given the difficulty of drawing three- and two-dimensional projections, not all hypersolids have been equally successful in literature and art. The most successful is definitely the hypercube, also called *tesseract*.

Among the images of the hypercube, the *Divine Cube of the Fourth Dimensions* of Flatland's Square, Henry Parker Manning's 1914 images (Fig. 3) became well-known even outside the circle of mathematicians. They represent two of the possible projections of the hypercube in three-dimensional space.

Manning's images are what Theo Van Doesburg uses in his four-dimensional architectural projects. The magazine *De Stijl*, founded by Theo Van Doesburg and Piet Mondrian in 1917, reprinted in 1923 an article by mathematician Henri Poncaré entitled *Pourquoi l'espace a trois dimensions?* [20] with the sentence *De Beteekenis der 4e Dimensie voor de Nieuwe Beelding* ("The significance of the 4th dimension for the New Design-Plasticism", the latter being the artistic movement to which

Fig. 2 U. Boccioni, *Forme uniche della continuità nello spazio*, 1913. Museo del Novecento, Milano © Comune di Milano—tutti i diritti di legge riservati

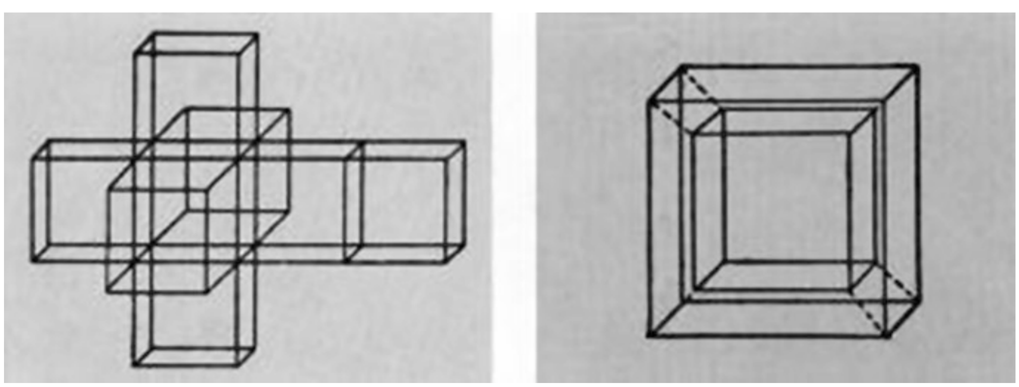

Fig. 3 H. P. Manning, *Hypercube*, 1914

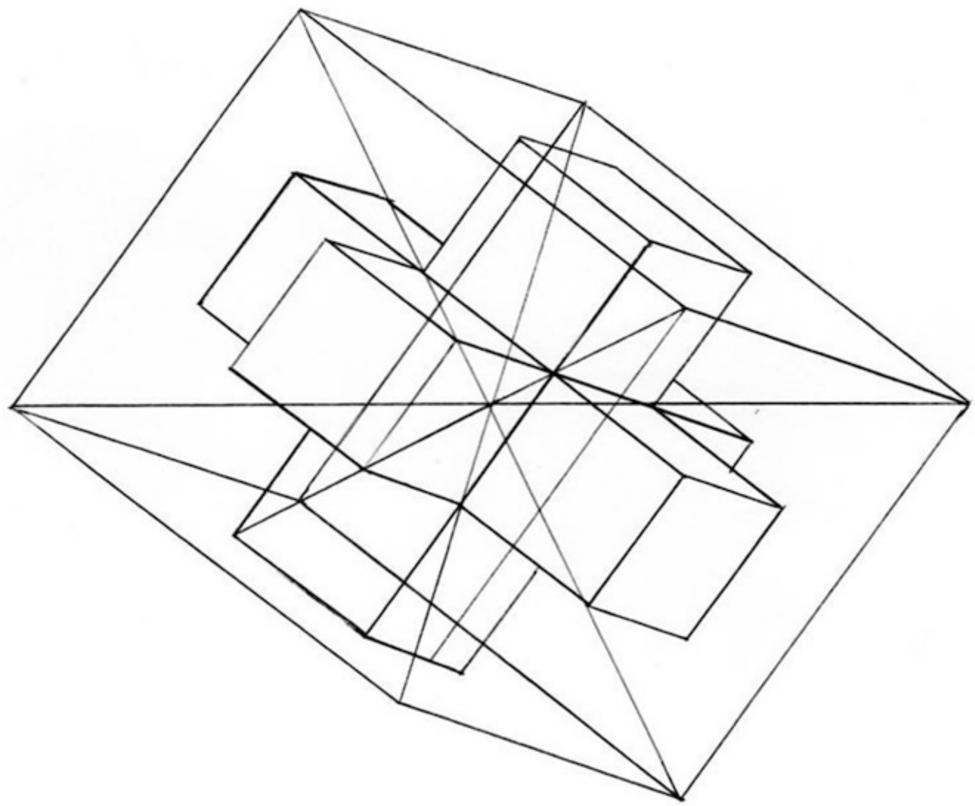

Fig. 4 T. M. Villarreal, drawing, based on T. van Doesburg, *Une nouvelle dimension pènètre notre conscience scientifique et plastique,* 1927 [21]

Mondrian and Van Doesburg had given rise) inserted as a preface. In 1927 Van Doesburg published in *De Stijl* a drawing of the four dimensional cube with the phrase *Une nouvelle dimension pènètre notre conscience scientifique et plastique* ("A new dimension penetrates our scientific and plastic understanding") (Fig. 4) [21].

Between the 1930s and the 1960s, with a few exceptions, interest in the geometry of the fourth dimension declined, both in the mathematical and artistic fields. One of those exceptions is Salvador Dalí, whose painting *Crucifixion (Corpus Hypercubus)* is from 1954.

A few years earlier, both Poincaré's research on topology and Riemann's research on non-Euclidean geometry, and the publication in 1947 of Coxeter's book on polytopes had aroused the interest of an Italian painter living in the USA and of a North American poet. They will create a kind of synthesis between Euclidean and non-Euclidean geometry, multi-dimensional space and topology.

The Moebius Strip

Upon a Moebius strip
materials and weights of pain
their harmony

A man within himself upon an empty ground.
His head lay heavy on a huge right hand
itself a leopard on
his left and angled shoulder.
His back a stave, his side a hole into the bosom of a sphere.

His head passed down the sky (as suns the circle of a year).
His other shoulder, open side and thigh maintained,
by law of conservation of
the graveness of his center,
their clockwise fall.

Then he knew, so came to apogee
and earned and wore himself as amulet.

I saw another man lift up a woman in his arms
he helmeted, she naked too, protected as Lucrece by her alarms.
Her weight tore down his right and muscled thigh

but they in turn returned upon the left
to carry violence outcome in her eye.
It was his shoulder that sustained, the right,
bunched as by buttocks or by brests,
and gave them back the leisure of their rape.

And three or four who danced,
so joined as triple-thighed and bowed and arrowed folk
who spilled their pleasure once as yoke
on stone-henge plain.
Their bare and lovely bodies sweep, in round
of viscera, of legs
of turned-out hip and glance, bound
each to other, nested eggs
of elements in trance.

This poem was written by US poet Charles Olson in November 1946 [22]. Corrado Cagli was an Italian artist who was interested in the topological surface of Moebius's strip, so much so that in 1946 he produced a drawing dedicated to the surface (Fig. 5) and a painting entitled A Moebius (Fig. 6). He was also a friend of Charles Olson's. Cagli's interest in the Moebius strip, in non-Euclidean geometry, and the fourth dimension of space began in the pre-war years, as early as 1939, when he arrived in the USA fleeing the Italian racial laws. He had always been in

Fig. 5 C. Cagli, *Anello di Moebius,* 1946, 51 × 33 cm, India ink on paper, private collection—courtesy Archivio Corrado Cagli, Rome

contact with other Italian and French artists and, when he held his first exhibition in the USA, he became part of the American cultural environment.

He enlists as a volunteer in the war, participates in the Normandy landings, and follows the front through Europe until he assists at the liberation of the Büchenwald camp on 16 April 1945. He will bring back a series of drawings based on what he saw in the concentration camp. Works of great realism and impact (Fig. 7).

The war had interrupted his relationship with Olson. On his return from the war, contact between the artist and the poet resumed. The publication of Coxeter's book comes at the right time. Although non-Euclidean geometries, the fourth dimension, topology and the Moebius strip are not entirely related topics, they are certainly linked to a concept that interests any artist: the idea of space. Cagli always cultivated a parallel interest in abstract and geometric forms in addition to his interest in figurative art. This led him to learn about and read books on mathematics, becoming interested in Riemann's geometry and in topology. In 1947, with Coxeter's book,

Fig. 6 C. Cagli, *A Moebius*, 1947, 50 × 80 cm, oil on canvas, private collection—courtesy Archivio Corrado Cagli, Roma

he discovered four-dimensional shapes. He was fascinated by them as he was fascinated by the Moebius strip, invented (or discovered) by German astronomer August Ferdinand Möbius in 1848.

In Cagli's mind, topology, the Moebius strip, the four dimensions, and the new non-Euclidean geometries became the elements of a renewed interest in geometries and mathematical surfaces. In particular, Cagli suggests that Olson read Coxeter's book. They discuss the Moebius strip and Olson asks Cagli to contribute with his drawings to a small book in which he will include some of his poems. These include the final version of *The Moebius Strip*. This short book is published in 1948 under the title *Y & X*. It includes five poems by Olson and five drawings by Cagli (Fig. 8) [23]. In 1948 Cagli returned to Italy.

Carlotta Castellani wrote in *Corrado Cagli e Charles Olson: la ricerca di nuovi linguaggi tra esoterismo e geometria non euclidea* ("Corrado Cagli and Charles Olson: the search for new languages between esotericism and non-Euclidean geometry"): [24].

"In order to express a multidimensional reality of complex simultaneity through his compositions, Olson undertakes a radical revolution in his writing, disrupting the composition of verse and the organization of language."

The first attempt to make his poetry into a spatial field is found in *The Moebius Strip* written in November 1946 but published as an introduction to Cagli's catalogue on the occasion of his exhibition at the Knoedler Gallery in New York in March 1947, and inspired by Cagli's drawing of the same title. Olson translates the distortions in place when projecting language onto a hypothetical Moebius strip:

Fig. 7 C. Cagli, *Bambino nel campo di concentramento*, drawing, Büchenvald, April 1945, courtesy Archivio Corrado Cagli

'The distortions + movements are intended to force language to do a like job in its dimensions as a painter would operating on a strip'".

Frank Moore recalls Cagli's constant interest in this geometric form: 'Cagli was obsessed with the enigmatic shape which had only one side and one edge and yet occupied physical space. He wanted to make painting on it . . . Cagli made drawings for Olson's words, Olson wrote words for Cagli's drawings. The poem about Moebius strip was one.'

The equivalence between Olson's poem and Cagli's drawing is acknowledged by the painter himself, who states, referring to Olson's poem: [22] "It seems to me that there is something very mysterious going on if, in the field of dimensions, drawings turn out to be poems and poems blow back sudden changes to the source of the drawings."

Starting from the assumption that it is almost impossible for anyone—scientist or artist—to visualize the fourth dimension, Olson perfectly described Cagli's way of proceeding by taking up Poincaré and Coxeter's analogy and reminding us that once

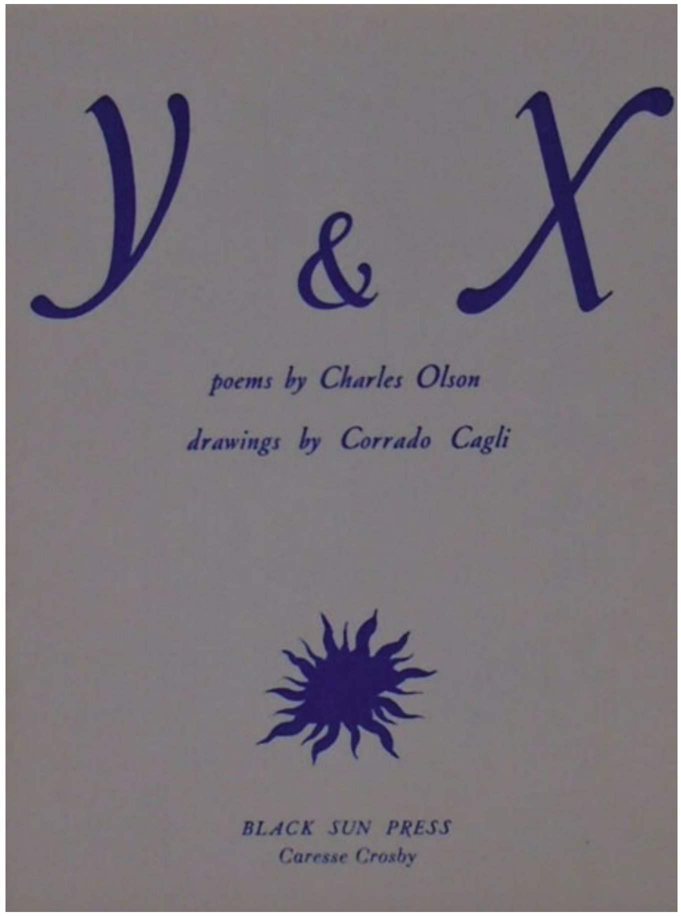

Fig. 8 Cover of the volume *Y&X drawings by Corrado Cagli/poems by Charles Olson*—courtesy Archivio Corrado Cagli, Roma

we understand the analogy between one-dimensionality and two-dimensionality, between two-dimensionality and three-dimensionality, it is possible to contemplate the analogy between three-dimensionality and four-dimensionality. The resulting image seems to describe a field of opposing forces in continuous tension, something similar to what Charles Olson was trying to convey through words. That these reflections were at the heart of the Italian painter's activity is also indicated by the works presented the following year in his personal exhibition in Rome, at the Galleria del Secolo (Fig. 9).

As Cagli recounted in his introduction, May 1949: "By drawings of the fourth dimension I mean those, including mine, which obey the spirit and optical taste of the projective that Donchian employed to represent fourth dimensional solids", [25] for the creation of these works the artist had been inspired by the four-dimensional solids of the self-taught mathematician Paul Samuel Donchian, which he had been able to see in Hartford, Connecticut."

The exhibition will move to the USA in December 1949 at the Watkins Gallery of the American University in Washington under the title of *Drawings in the 4th Dimension*, with a lecture by Charles Olson and a note by Cagli [26] "When I speak of drawings in the 4th dimensions I am referring to those of my own which obey

Fig. 9 C. Cagli, *Catalogue of the exhibition,* Galleria del Secolo, Roma, (1949), courtesy Archivio Corrado Cagli, Roma

the optic spirit and taste, which the mathematician Donchian has expressed in his projections of solids in the 4th dimension. To be elementary that which appears as a cube in three-dimensional space, will in the space of four dimensions, take the form of a hypercube. Since the antithetic space significance of these two solids is understood it becomes possible to see them as measures of two different pictorial system-the cube as the rule and the measure of all paintings in three dimensions, the hypercube as the rule and the measure of paintings in the 4th dimension . . .

On a page of two dimensions, a drawing in the 4th only takes an allusive, not representational force, and I strongly suspect that we will not be able to adventure into the slightly explored field of the n-dimensions until we are prepared to give up both the frame and canvas . . .

I have, within the limits of my research, made some experiments with the Moebus (!) strip and it offers a pure shape and a continuous surface no less suggestive that the circle, no less impressive that the sphere."

The exhibition featured *Eleven Hypercube Drawings*, among others. There should have been some of Donchian's models in the exhibition, and on this subject Davide Colombo writes in his extensive essay *Non-Euclidean Geometry and the Fourth Dimension in the Intellectual Exchange between Charles Olson and Corrado Cagli* [27]. "In a letter dated December 8 1949, Cagli complains about the decision to exhibit Donchian's models of four-dimensional solids because they are too theoretical and programmatic, citing some provocation and risk of confusion... The doubt arises that Cagli's reluctance may be due to the risk that his drawings will be judged as a direct transposition of Donchian's models, thus denying their autonomy and artistic value. In the end, as Olson later recalled, the Donchian solids were not exhibited because they were not in good condition." The exhibition was then taken to *Black Mountain College*, where Olson had been invited by Josef Albers. Olson would be head of the famous College Olson from 1951 to 1956.

Colombo adds: [28] "Neither Olson nor Cagli had specific mathematical and geometrical training: they rely on intuition for their perception and understanding of mathematical concepts, although this does not negate the careful study of mathematical principles. In Cagli's case, these are also influenced by some explanations on non-Euclidean geometry given by his brothers-in-law Oscar Zarinski (husband of his sister Iole) professor of mathematics at Harvard, and Abraham Seidenberg (husband of his sister Ebe), professor of mathematics at Berkeley University and then at Harvard, and by other mathematicians he met, as he remembers in a letter of May 12, 1946. But it was the encounter with Donchian's models that provided Cagli with the inspiration to radically evolve his research."

Donchian is mentioned by Coxeter in *Regular Polytopes* [29]. He also writes a brief biography of him and recalls that section 13.2 *Orthogonal projection onto a hyperplane* [30] of his book is directly inspired by Donchian's models, two of which were photographed and included in Coxeter's book. Tables IV on page 160 and Table VIII on page 273 show the {3, 3, 5} and the {5, 3, 3} polytopes, respectively. (Fig. 10).

"Paul S. Donchian was an American of Armenian descent. His great-grandfather was a jeweler at the court of the Sultan of Turkey, and many of his ancestors were oriental jewelers and handcraftsmen. He was born in Hartford, Connecticut, in 1895. His mathematical training ended with high school geometry and algebra, but he was also interested in scientific subjects . . . He made a thorough analysis of the geometry of hyper-space. His aim was to reduce the subject to its simplest terms, so that anyone like himself with only elementary mathematical training could follow every step . . . Their constructions required all the patience and delicate craftmanship that could be provided by his oriental background . . . To quote Donchian's own words: 'The models are fortunately fool-proof, because if a mistake is made it is immediately apparent and further work is impossible.' In 1934 the models were exhibited at the *Century of Progress Exposition* in Chicago and at the *Annual Exhibit of the American Association for the Advancement of Science* in Pittsburgh. He died in 1967."

Coxeter met Donchian in Chicago at the show and photographed the models he would publish in his book. Together they will write a July 1935 article *An n-Dimensional Extension of Pythagoras' Theorem* [31].

Returning to Olson and Cagli, "Just as the scientist had restored through an open form of sculpture the idea of the fourth dimension, in the same way the artist sought to achieve this representation by analogy in the two-dimensional space of the canvas." [32].

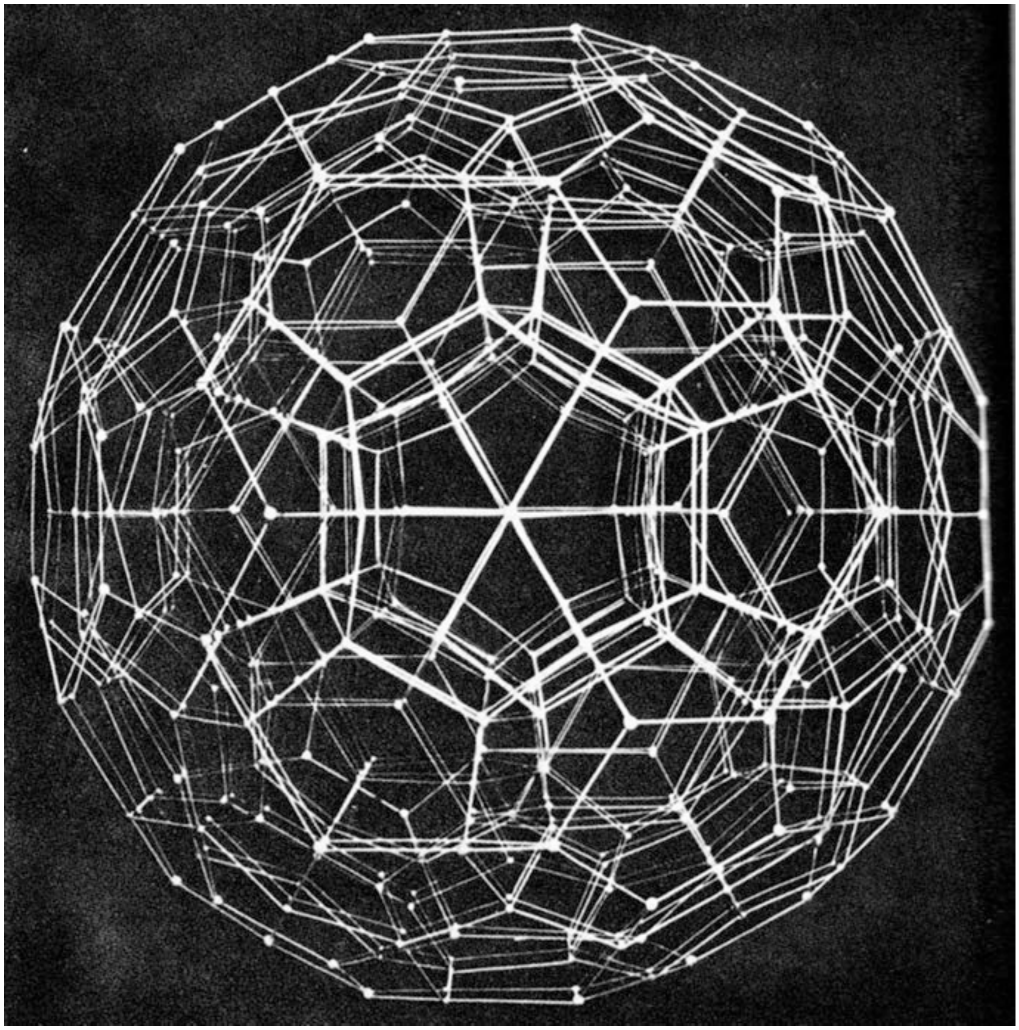

Fig. 10 {5, 3, 3} Model of P. S. Donchian, photo by H.S.M. Coxeter, [1], p. 272

In 1946, the American poet formulated his first ideas about the possible application of the new mathematical theories to poetry. The geometric model offered the possibility of inserting an object belonging to a Euclidean space into a non-Euclidean space. Castellani adds that "It is difficult to understand exactly what the application of these geometric rules means in terms of language." [32].

Olson recognized in his friend Cagli's art the most advanced experimentation with the fourth dimension. In the letter written by Cagli to Olson on December 9, 1946, there are some important considerations about the poem on Moebius and about their common purpose: [33].

"Upon a Moebius strip. I think you are going strong. The all business is wonderful. The poem is up to your best poems, isn't it? And brings me a new wind of inspiration, it seems to me that there is something very mysterious going on there if, in the field of dimensions, drawings turn out to be poems and poems blow back sudden changes to the source of the drawings. (...)any time you show me another poem another door is opened and it looks the way it should look: as the initial point, the beginning, the primordial way of thinking and feeling. (that would be in terms of the tarots: the *Bagatto*) [24]. And I feel very strong about us now, as we have found together a mine of gold."

Colombo notes: [34] "The great novelty that projective geometry and, in general, modern physics and science can offer to Cagli and to art is the possibility of a type of abstract thinking that sees the world as a whole and, at the same time, makes sense of the structure and function of the world as a whole; projective geometry gives the possibility of imagining a completely new space and world, that is, of finding new (and better) solutions at the level of art and thought and at the level of morality."

In his 1949 lecture at the *Watkins Gallery*, Olson traces an intense and fruitful intellectual and human history. The idea is to explain the new concept of space, on which Cagli and he are working, which leads to a new art and a redefinition of man, to realize a morally new humanity [35]. "What I want to do tonight, to justify my appearing before you, is to illuminate for you in what a new conception of space (which is, I think, what Cagli & I keep working towards) leads toward a new art & thus toward a redefinition of man, accomplish, in the moral sense of a new humanitas."

Mark Byers also talks about this *redefinition of Man* in *Charles Olson and American Modernism* [36]. "While Coxeter noted the *psychological value* of the models (since they offered at least a metaphorical visualization of the fourth dimension) Olson thought much more of them. 'What is involved here, he suggests, is something which both science and art have long been capable of, the act of taking a point of vantage from which reality can be freshly seen."

Colombo adds at the end of his article: [37] "Cagli's own research method responds well to a trend that gradually emerged during the 1950s and which saw science not only as a kaleidoscope of new images and aesthetic suggestions that risk becoming mere decorative motifs if not supported by reasoning and solid foundations, but as the bearer of an experimental and operational methodology and approach, of a more open vision."

Corrado Cagli died in Rome on March 28, 1976. In 1978 I visited an exhibition of his work at the *Ca′ d'Oro* gallery in Rome and was struck by an untitled painting that was accompanied in the catalogue by Charles Olson's poem on the Moebius strip. (Fig. 11) I wrote about the Moebius strip and about some of the artists who had been interested in that form, first of all Max Bill and Maurits Cornelis Escher; the article was published in 1981 [38] in the magazine *Leonardo*, then printed by Elsevier. It was edited by Frank Malina, a North American kinetic artist who had moved to Paris from the USA after leaving his job as a rocket engineer. I mentioned it again in 1983 at a conference at the *School of Epistemics* of the University of Edinburgh in November 1981, where Frank Malina was one of the speakers; however, he died the day before the conference opened [39].

At that time I started to work on my film on the Moebius surface, [40] in which Max Bill and the works of Escher were involved, so I phoned Cagli's atelier and made an appointment to go and film the painting I had seen. The atelier was in

Fig. 11 C. Cagli, *Without title*, oil on canvas, (1947) Collection Ebe Cagli Seidenberg. Frame from the movie *Moebius Strip* [40]

Rome, in Via *della Fonte di Fauno 12* (near the Circus Maximus) and is now the headquarters of the *Cagli Foundation*. I remember the day very well, it was March 16, 1978. When I entered the atelier, I was informed that Aldo Moro, president of the *Democrazia Cristiana* (Christian Democrats), several times minister and prime minister of the Italian government, had been kidnapped by the Red Brigades. He was killed on May 9, 1978. One of the great tragedies of the Italian Republic.

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