A Study of Ornamentation
by
Max Dehn
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I.
Method in Applied Mathematics

The following investigation of ornamentation can be considered indeed as an area of applied mathematics to which little consideration has been given until now. For that reason, I would like to set forth first a few remarks on method in applied mathematics.

How does mathematics enter into the description of natural phenomena? We shall take as an example a falling stone and try to describe it as a mathematical sequence. We begin with the simplest: the stone moves, i.e., the body changes its position in space or relative to a particular standard body. The stone falls, that is, over time the distance between a suitably chosen point on the standard body (the Earth) and a suitably chosen point on the (falling) body gets smaller and smaller. The stone *falls* faster and faster, that is, during the time between successive swings of a pendulum the distance covered by the body increases. Still missing here is a mathematical expression for "pendulum swings" —perhaps "the extremities of a certain periodically repeating standard movement." And so on. If one chooses not introduce intermediate concepts, the purely mathematical representation becomes ever more complex as it penetrates a phenomenon more deeply. If one does introduce such concepts, one can then indicate the following assertion as the result of this analysis: the distance fallen varies as the square of the elapsed time.

Occasionally, through calculation, mathematical description enables us to make further assertions about natural phenomena. In many cases these *conclusions* resulting from mathematical analysis are not greatly significant. They do not deepen or refine the description substantially. There are, however, cases where original analyses by means of purely mathematical solutions, deliver a wealth of knowledge which inspires us and fills us with deeper satisfaction.

In these cases, the description is generally such that one associates the group of phenomena with an especially simple mathematically describable process, one which cannot be directly observed—called a law—from which one not only derives, in reverse, the mathematical description of the observed phenomena, but also many others that have yet to be observed. The Newtonian law of attraction as a description of the accelerating action of heavenly bodies upon one another, from which follow all the observable reciprocal movements of heavenly bodies, offers a particularly beautiful example of this.

Of course there is a great endeavor in other areas also to arrive at a point where through mathematical analysis the amazing instrument of the calculus—miraculous in the feeling of many—can be applied. But this often leads rather arbitrarily to the imposition of mathematical structure upon phenomena, as was the case frequently in the past, for example, with regard to the national economy. Here it can be difficult to be content in the first place to analyze mathematically with the greatest possible precision economic phenomena, conditions and their modifications. And it is no small task, because it is necessary besides to have at hand not only all possible mathematical concepts, but also many indeed must first be developed in certain particulars. It may be then that this close analysis will show that economic phenomena are unsuited to calculation.

Every mathematical analysis of reality is a bit forced. For this reason it is also so dangerous to accept without further (investigation) the results of possible mathematical conclusions. One will therefore test them in reality. With this test contradictions often result between reality and results which reveal that mathematical representation is only an approximation.

The most faithful possible mathematical analysis does not always yield a good description. If I want to represent the impression of evening light, expressions such as "somber banks of cloud with beaming golden edges and a few radiating every color and above small clouds floating in the blue" *produce* a much stronger effect than were I to make a precise mathematical statement. This non-mathematical description functions directly through resonance. One feels something hearing words such as "somber," "beaming," "radiating," "floating," etc., but only a little from those like "few" or "above" which play a role here and which are easily available in mathematical analysis.

It is self-evident that every description can have an effect only on one who understands the descriptive language. Otherwise, resonance is, of course, impossible. Mathematics is an international language easily transmitted through symbols. In this connection, it is thus already an advantage, if one can give a penetrating mathematical analysis. The following is more fundamental: mathematical analysis is neither the only nor, as noted above, always the best description of a phenomena. But the transmission within me from yesterday to today and to tomorrow, the transmission from one human to others is the more secure the more it is mediated by mathematics. For example, knowledge of the number of legs on an insect is, as far as number is concerned, utterly primitive mathematics. What a leg is, however, is not so obviously mathematical, or, if you will, subject to scientific analysis. Accordingly, there can be doubt about what phenomenon is meant, because one can just as well start from the mere shape as from the anatomical, ontogenetic, phylogenetic or the functional importance of these members. The more precisely and securely one aims to represent the object, the more one will, I believe, have to use comprehensible mathematical concepts.

Exact representation, however, does not exhaust the task of science. Along with that comes the *cognitio causarum*, the manner of viewing which creates a series of phases, out of aspects (observed) at various times, in which each phase is the cause of the following one. It is characteristic of this process that the different aspects are not merely different in the way that sequential microtomic cross sections follow one another but rather that one must jump from one aspect to a different one. This part of science is distinguished by the fact that it represents that which cannot be observed, namely, causes. This is precisely where science is especially fruitful. Otherwise, one dives again into mathematics. But then one has that very peculiar feeling of conclusiveness, the feeling that the individual leaps between phases can be made smaller and smaller at will. Outside mathematics that is not the case. The *cognitio causarum* does not belong in the realm of applied mathematics.

Now that we have, in the foregoing, compared mathematical analysis with different representations, we must busy ourselves for a bit with mathematical representation itself. Something very simple in mathematics, which one can assert about any phenomena whatever, is the relation of equivalence between something and another more encompassing thing. All congruent ellipses (the same with regard to motion), all Ellipses, all conic sections, all algebraic curves, all continuous curves, all coordinates of the points of two linear continua, all coordinates that constitute a series, these are analogous to the series beginning with Veronica chamaedrys and leading across species, families, the totality of the dicotyledons, of flowering plants and so on right up to the totality of all organisms.

Now the beginning of an analysis consists frequently in the search for such equivalent series and thereby obtain a *classification* of phenomena. One may find an equivalence, however, in various ways. For example, if I find that a man whom I see now to be the same man whose acquaintance I made yesterday in the train, I do not need to be aware of any particulars in discovering this equivalence. But perhaps I have the obligation nevertheless to clarify for myself concurring particulars or to express differences. Sometimes these particulars are analyzable mathematically, but they would not be analyzed in this manner.

It is already a scientific achievement if, for any large group of phenomena, a “rational” system of equivalent series, a rational system, is found and set forth. For this system (to exist) there is no need to specify any determining equivalences. It can also operate as something reasoned, i.e., something articulated abstractly. Often determining equivalences are given for the system, often a scientist will try instinctively to represent equivalences in such a way that they can be grasped mathematically, and, finally, they may be represented deliberately as mathematical. In this case, mathematical matters present themselves as mere equivalence in relation to something—quantities perhaps, or shapes, or relationships between numbers and shapes—in other words what are called functions in the language of mathematics. In this way, through systems of series of equivalence, through ordering, and through the scientific description of groups of phenomena, one arrives at the application of mathematics.

Whenever one pursues applied mathematics, one has the same problem, namely, the one already mentioned of finding a judicious way to introduce mathematical concepts. If you wish to apply mathematics to a national economy, it makes no sense to tackle the classification of settlements by the average width of their streets, just as little as it would in a mathematical botany to attempt the approximation of leaf forms by means of algebraic curves. As a particularly simple example for the method let us treat the problem of mathematically investigating an isolated shape. We will consider the task of analyzing mathematically the shape of the Norwegian coastline. To this end perhaps one will designate a polygon such that the coastline nowhere extends farther than 30 km from the polygon and determines the form of such a polygon by means of number, size and the angle at which the sides meet. If one does the same for a maximum distance of 3 km and one of 300m, one will recognize in the growing number of polygon sides the fine detail of the coastal progression, the whole realm of its coves and inlets. Thus one can also describe for any section of coastline for which, similarly, the shortest side of a polygon will be established with a corresponding maximum distance, while of course it makes no sense without such or similar stipulation to speak of the length of the coastline. This appears to be a “reasonable” analysis of the coastline. Whether it satisfies any practical requirements is another question.

The analysis of the shape of a coastline obtains its worth, of course, through the comparison with the same analysis of different coastlines. If the difference in shape thereby becomes clear through the variation in the corresponding numbers, the analysis will be regarded as “reasonable.”

A completely different situation is under consideration if one wants to analyze *groups of phenomena*. In the first place, one wonders by what means a group, i.e., a multitude of correlated phenomena or individuals will be characterized. That will always happen no doubt by means of “equivalences.” But these can be of very different sorts. Thus arise groups of men all inhabiting the same country, all the heavenly bodies in the solar system, or all living organisms. The equivalence which defines the first group is mathematical in kind: humans are numbered among others in the group in virtue of their location. The second group can perhaps be defined thus: all heavenly bodies belong to the solar system whose observable movement is influenced by the gravitational force of the sun. This is already no purely mathematical definition. One could also define the bodies of the solar system by saying that they are exceptionally much closer to one another than bodies not belonging to their group and that the sun and the earth belong in the group. This kind of definition is by far the simplest that one can give for the three groups: between two living organisms others can be inserted, so that in this way an uninterrupted passage between the two organisms results, and the lion as much as the diphtheria bacillus is a living organism. There are indeed many different definitions for the group of living organisms, but if they are not obviously inadequate, so they are artificial and only provisionally incontestable. This same condition holds in still greater measure for the subgroups of animals and plants. Here there are organisms whose membership in this or that group one can dispute. And yet the domain of animals and the domain of plants are completely unequivocally defined, right up to those "border areas" whether one starts from the lion or the earthworm, or on the opposite side from the basswood or the edible mushroom.

Now once the group has been more or less consciously established, the researcher seeks to discover further equivalences for it. Here one finds astounding realizations, for example concerning the similar appearances among all living beings and ever more equivalences for narrower subgroups. Every equivalence that still seems so unimportant among a large number of individuals is significant because it must, so one thinks, evidence a (causal) relationship between all things shared by all individuals in the group. But it is often difficult for a group of undoubtedly related appearances to find a sure, mathematically comprehensible, equivalence in the most general sense. And yet one would like to characterize in particular those individuals or (phenomena) which merely in virtue of the quality of association show themselves to belong together. But that correlation need not have consequences that are easily grasped. Without a doubt, for example, the so-called German Jews are a group whose members already belong together by means of multiple kinship ties. But until now characterizing them by means of a different equivalence has not succeeded. In such a case, how can one seek to further characterize the domain?

In many cases, perhaps also in the example just cited, one may succeed in finding "types" —certain particular individuals having a series of characteristic qualities. There could also exist "ideal," not at all realized, individuals having this series of qualities with whom one can associate all individuals in the group such that the difference is very small between one individual and an associated type. Of course, this positing of types only makes sense if the number of types is substantially smaller than the number of individuals in the group and, further, if the individuals belonging to the complete group cannot be so well associated on average with the types as with the individuals of the group. For example, supposing one could find 500 types of Jews in the group of about ten million "German" (non-Hispanic) Jews, and that the group could not be made as broad as that of all white-skinned people such that on average white people would fit the type just as well as German Jews, then an essential part of the analysis would have succeeded. At the same time it is entirely thinkable that neither the system of characteristic qualities, nor the divergence of any individual from an associated type, is in any way mathematically comprehensible (like the distance of a point of coastline from the nearest side of the inscribed polygon). But, of course, the assertion is on safer ground, if it succeeds in establishing analytically, perhaps by means of mediating forms, the types and the group's deviation from them.

So what now about the analysis of ornamentation that we wish to describe? To begin with, the demarcation of the group of phenomena: and, of course, the most suitable [approach] is again the definition by means of association. The figures on prehistorical vessels are ornaments, but so are Greek acanthus leaves and the paintings on shields from New Guinea. One passes from one of these human labors to another and everything one touches is undoubtedly ornament. By contrast, there are symbolic figures fashioned by primitive peoples that are without doubt not ornaments and on another side are the Greek statues and Netherlandish paintings which are also certainly not ornaments. With these the domain of ornamentation is in some measure defined. But even here there are difficulties with boundaries.

Thus, one may be in doubt whether pleats in the garments on Greek statues should be referred to as ornament or as idealized representation. There is in fact a seamless [my pun. sorry] transition from ornament to every artistic work. However, we need not remain content with the definition by means of association. Through our own analysis we shall come upon a deeper definition based on characteristic "equivalences" for all forms of ornamentation. Then the domain limits will interest us because there, alongside the purely ornamental, a different element advancing the study of ornament will take on intrinsic significance.

What now pertains to this analysis, [is] the search for what is characteristic; so we shall try to assemble the various types around which all the observed forms group themselves. It is characteristic of these subgroups to be mathematically comprehensible and for a very special reason: *the human who creates ornaments and contemplates them has intuitions which in an essential part agree with the intuitions of the human who creates or contemplates mathematics.* For this reason, ornaments have a mathematical aspect and are mathematically well analyzable. However, in this analysis, the emotional origin must be steadily kept in mind so that we do not interpret our findings as having more mathematics in mind than the maker of ornaments intuited. — What we said about the study of ornament as such, is also valid, by analogy, for acoustical ornaments, for music as well as for architectural creations. We shall in passing also touch on these two fields.

And thus we shall begin with the simplest types, together with the simplest emotions, that stand in connection with these creations.

II.
Mathematical Analysis of the Simplest Ornaments

Very old things fashioned by human hands exhibit ornaments. Thus, we find decoration scratched or impressed on clay vessels from the Stone Age which are more than ten thousand years old. Soft clay lends itself very well to the first creation of interruptions in the smooth surface and whether the clay is dried or in later periods fired, it preserves the shape of these interruptions almost imperishably. Such an ornament consists more or less of parallel horizontal (i.e., parallel to the bottom surface) rows of similarly shaped equidistant "points." We have here, thus, *two equivalences* which are connected and make an appearance as ornament. But if one considers more closely this simplest of all ornaments and its preparation, one recognizes yet another element: the maker must "work out" i.e., choose the distance between two points such that it will reproduce the given curve running around the vessel. So one has thus in practice a *geometric problem* to solve. The solution affords satisfaction as the solution of a reckoning assignment with a test for the correctness of the solution. The potter measures, perhaps with a cord, divides up, and when he's done, throws the pot and is pleased to find the evenly distributed points resulting from his craft. If one looks at the pot, however, without throwing it [oneself], one has the feeling that the row of points continues "arbitrarily." Thus we have here in the production of ornaments a realization of an *arbitrarily continuous operation*. No doubt this makes an impression, even if here it is not quite yet a strong feeling, for the infinite.

Very often on ancient vessels one finds not only rows of single points but rather perhaps two rows arranged so that every point in the lower row is impressed between two succeeding points in the upper row. With that we have a very simple functional operation: for every sequential pair in the upper row, a new point is assigned. The pair in the one row represents the *independent variable* (the given), the pair in the second row the dependent variable (what is sought). This functional operation will be completed when the ornament is finished. The [operation] has a very special quality: if one takes up the same operation with the second row, the original one results with a displacement toward the bottom. Surely one has a sense of satisfaction with the completion of the ornament in the contemplation of an event conforming to a rule, or one could say to a higher rhythm, as indeed it was a rhythm when by one point following regularly after the other, the first row was again expressed. The second row also seems to evidence a certain disagreement with the first row: “No, I will not simply put the points directly beneath the existing points. On the contrary, [I shall] place each one, for a change, in the middle of two. The contradiction of the contradiction however, yields the original. This contradictoriness plays a large role in ornamentation.

Our analysis brings us in addition to recognize a similarity between ornaments on clay vessels and those on completely different materials, even non-visual products of human activity. Australian aboriginals, to whom the potter’s craft is unknown, make objects—shields, for example—adorned with feathers arranged just like points on the vessels. Moreover, we find such elements in domains not customarily brought into direct connection with ornamentation, for example in the dance, and in music accompanying the dance. In the dance and dance music, again, the essence lies in the equal distances—here, in repeating intervals of time in the stamping of feet and the drumbeat. If we have two drummers, one of whom strikes exactly between two beats of the other, we have thus the same mathematical elements as above with the rows of points on clay vessels. The opposition between the second and the first drummer here becomes very significant. With dancing and drumming, the impression of limitless progression becomes even stronger. It may become overpowering, as when the dancer or the drummer is so enthralled that he cannot stop until he becomes physically incapable.

But drumming also affords more complicated structures for analysis, for example in triplet drumming where two weaker beats between the downbeats follow each other and the downbeats at the same interval. Here we have a more complex network of equivalences and antitheses. “If in certain tribes, as ethnologists report, several different rhythms, drumbeats, are executed simultaneously by different people, a situation arises that is already close to a mathematical operation. The joy in the regular recurrence of the moments in which the stressed beats, the downbeats of the various rhythms, strike together corresponds to the joy of the fledgling number theorist who notices how multiple [repeated?] series of numbers are embedded in one another.” {see M. Dehn, Das Mathematisches im Menschen, “Scientia” Sept. 1932, S.123.}

The [cultural practice] most highly developed in this direction is certainly change ringing in England. Using from six to eight church bells of completely different sounds, ornaments are fashioned by altering according to definite instructions the order in which the bells are rung. With each “touch” all the bells sound, but every time a few or more pairs of bells are rung as directed they are exchanged with one another. These directions were devised by master bell ringers. Undoubtedly, the ringers—one for each bell—find great satisfaction in discharging these directions all the way to their liberation, the triumphant conclusion when the bells again sound in their old-established order. It is difficult for the common listener to recognize and enjoy the controlling regularity and this acoustical ornament, often lasting more than an hour, can seem like torture. One can even say that for the average listener no controlling regularity exists, because regularity implies anticipation which can only be enjoyed when the anticipated outcome actually arrives. So we have here the acoustical realization of combinatory operations [accomplished] indeed directly by particular substitution groups.

There are also (visual) ornaments which exhibit the association of several rhythms, such, for example, as the two to five rhythm in the inscription of a circle within the Pentacle. The ornament on clay vessels shows the simplest operations with which we began our consideration. But there are hardly such various realizations among *visual* operations as we learned about from just one acoustic [realization]. The two-dimensional surface as the basis for ornament stimulates a different kind of development than the simple course of time as the basis for acoustical ornament. We looked at the above-described simple ornament on a clay vessel. If we continuously connect by a straight line two points in the top row with the point between them in the lower row, we obtain a zigzag line. This zigzag line tends to become a wavy line by an *evening-out of the discontinuity*. This "evening-out" is something easily understood from muscle movements as much in the creating hand as in the observing eye. It is easier and more pleasant to draw a wavy line in one stroke, or to follow one with the eye, than a zigzag line. Of all possible changes in appearance this is an occurrence of the utmost importance. The process of evening-out, which the mathematician undertakes with his shapes, has different roots. It springs mainly from a [self-] assigned task: discontinuous forms should be drawn together by means of simple mathematical expressions. These [expressions] however, always depict forms without discontinuity.

Frequently, wavy lines are designated as a *representation* of waves of water, and one could get the idea that this ornament as well as many others originate in the imitation or representation of forms occurring in reality. We want here to consider this idea somewhat carefully. The occurrence of a representation is in itself already something very complicated: after all, how does a human being arrive at the representation of something from Nature? Further, how does it happen that he represents it more or less "symbolically" as he does in our case of waves of water, with the stroke of a wavy line? Why does he often make use of such representations as adornment or embellishment? Of course, we can only treat these questions cursorily here. To begin with the first question: one can be obliged to represent something from reality in order to communicate something to someone distant, or to posterity, perhaps by means of hieroglyphics. In this case, however, the receiver or later reader of the information must be able to understand these hieroglyphics, must know what is meant by the signs and symbols substituting for reality. Secondly, the representation can serve magical purposes; through the symbolic representation of of a wished-for reality, one supposes one can obtain the realization of what is wished for. Related to this is [the wish] to make the divinity visible. In this way, one has a position in which to venerate the Invisible and ask for the fulfillment of wishes. That which is represented in this way more often exceeds in large part what may be seen in the real world. In this manner, the divine, the superhuman, is symbolized, as shown, for example, in the many-membered images of Indian gods.

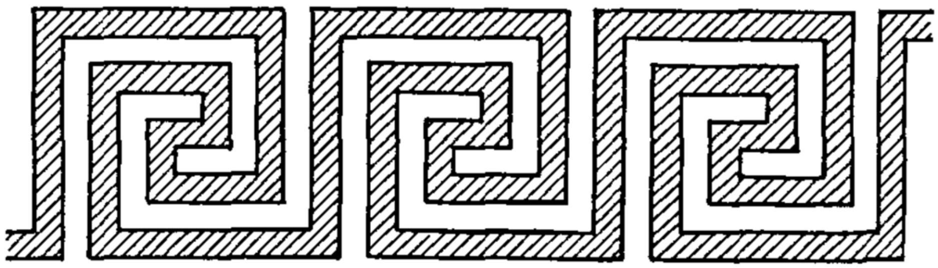
Thirdly, pure joy in creating can be an important motive: "I, too, can make a crocodile, a man, sea, and stars." Finally, Nature itself can inspire humans to imitate. What inspires can, again, be very diverse. It can be essentially erotic Nature, as in the form of the human body, or essentially rhythmic as in the regular branching from the trunk of a large tree, or many other sensuous things in which the erotic and rhythmical generally play a part.

In all the occasions for the representation of natural objects thus far enumerated here, the most important role is played by the representing human. Primitive man rarely tries to depict anything from Nature so accurately that the observer can mistake it for the natural object. The New Guinean heads of dead ancestors are most realistically executed, but they are only heads, and for this reason, arranged in the hut next to each other on a plank, they appear, of course, nothing like human beings. It is worth noting also that, in prehistorical depictions, two-dimensional representations--for this reason alone unrealistic-- appear to dominate. Most primitive depictions are very obviously governed by symbolism, i.e., in the act of representation, humans impose their own rhythms upon natural things and, hence, one can say that they practice applied mathematics. For example, the intuition of the zigzag line came first, before it ever became the symbol for lightning and, in the form of the zigzag, the original muscle dynamics are combined with the symbolic representation not just of the shape but also the power of lightning. Of course, real lightning is often not at all perceivable in this form. Very similar observations can be made about other simple depictions of natural objects: the double bow representing a bird in flight, or the double hook as the image of a palm tree. In general, too, in the [fully] developed technique of the artist, among the extraordinary number of common shapes unknown to him, perhaps sometimes known to him, the shapes felt as suitable for the reproduction of a visual impression are employed. The case in which the movement of the eye muscles in following an outline immediately engages the muscles of the hand in tracing it, is surely to be considered the extreme case. For an "immediate" representation, i.e., one independent of its own rhythm our impression of natural objects is, however, almost always too complex. In this inquiry, therefore, we will need to consider only secondarily the significance of Nature for the practice of ornamentation. We shall disregard the magical import altogether, for it influences the inner rhythm even less.

We turn again back to the consideration of ornamentation on clay pottery. If one has several rows of points one below the other and such that the points of one row always stand midway in the gaps between the points in the rows above and below, and if, as previously, two points are connected with a point between them in the next row, we will get a lattice of rhombuses, providing the rows all have the same distance from one another. Thus, from our original rhythm we have obtained something new, a division of the surface into nothing but identical parts. The division of the one-dimensional into exactly equal parts, the one-dimensional rhythm, connected with the contradictory and parallel rows, leads to a two-dimensional rhythm, something certainly not previously found among humans. The quality of a two-dimensional continuum gives birth to something new and this new thing, *the unintended*, is already to be denoted as a *Theorem*. The maker sees that what he intended, what proceeded from him, necessarily yielded something new. So one can say that the existence of the rhombus, or parallelogram lattice which comes to light approximated on the vessel, constitutes one of the first mathematical theorems ever encountered by a human.

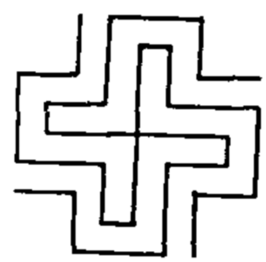
The lattice quickly leads the experimenting human to a further theorem. One very often finds on clay pots not only the simple rhombus lattice, but rather a division of the rhombuses into two different groups. If one rhombus is hatched [shaded], all the neighboring ones will not be. It now occurs to the potter that in this manner the entire lattice can be treated so that a checkerboard pattern emerges. He only has to take care that the rows running around the vessel have an even number of points. But if this condition is met, then he will manage his hatched and unhatched so that a hatched rhombus only ever bumps into an unhatched one. One sees how the checkerboard pattern emerges by applying to the rhombus lattice yet again the principle of opposition: every rhombus like its neighbor will be set against its opposite. The existence of the checkerboard pattern is already a really interesting theory, and we see that very primitive people were aware of it.

Such a division of a plane surface can also be produced by braiding. One hits upon the idea indeed that the checkerboard pattern is to be looked upon as a representation of a braiding sample. But the existence of this sample is not enough. The form must first come into consciousness before it can be illustrated. After that, the person looking at the sample would have to be so happy with it that he imitated it on his jar. But that is a very implausible notion. It is already not certain that the checkerboard pattern only ever occurred to people familiar with braiding technique. However, there are certainly many figures associated with the rhombus lattice or, especially, with the square lattice which originated through experimentation with this figure and in complete independence from the braiding sample, first of all the *Meander-Ornament*. This [figure] solves the problem of how to draw, between two parallel lines in a square lattice of dimension n, with the help of the lattice sides, a stripe (the Meander River) having itself the width 1 and [how to keep drawing] the stripe between the parallels until it fills [the square] with stripes of width 1. The solution is not at all a self-evident figure; for example, to start with, in preparation, for it to come out for any n, n must be an odd number and the distance [permitting] two full swings of the river, the period of the Meander stripes, must have the length $n + 1$.



Meander $n = 9$.

There are many other ornaments that result from experimenting (playing) with square lattices. I shall present but one which is perhaps not so well known. It is to be found on ancient Etruscan clay jars, for example in the Villa Julia in Rome. It is composed of 4 lines through the midpoint of a square having 8 units on a side. The lines go forth separately turning at 90 degrees and divide the square into stripes that everywhere have the width of 1 (on the jar in Rome and also on the statue standing nearby the lines are not extended into the small corner squares, perhaps out of aesthetic reasons). Without doubt here, real *mathematical problems* are being solved with these and similar figures. The figures themselves are to be viewed as mathematical theorems. Of course, inversely, not every mathematical figure is an ornament. It is required that a rhythm in the figure, a combination of equivalents, come to light.



Falisio-Nekropole
— 700?

We take up again the parallel rows of points running around the jar and apply to them a triplet rhythm, i.e., we mark every third row. In the remaining rows we connect the points of the two neighboring rows, as before, with a zigzag line, and the points of two rows, the one lying directly underneath the other and separated by a marked row, we connect with vertical lines. In this way, a six-cornered lattice emerges, a division of the plane into perfect hexagons. Such ornaments also are found on ancient vessels, e.g., on a potsherd found in Kristiansund from the early Stone Age and part of the collection of the Scientific Society in Trondheim [yes!]. One might compare this figure to the honeycomb on the basis of similarity and think that the ornament can be traced back to this natural object. But what was said above about the braiding model and the checkerboard holds true here. It seems to me that both figures simply originate from the productive capacity available in people themselves. I believe that if one likes to play with ornaments and by means rhythmic operations sees more and more new ones arise from them, one will come more easily to the view that the connection with natural objects as the genesis of many ornaments has only secondary importance. Moreover, we notice that vis-à-vis the method of applied mathematics set forth in the introduction, we have arrived here in good health. We had spoken there about the *cognitio causarum*, which is indeed of special importance for science, but does not belong in the domain of applied mathematics. Here, however, we were unable to abstain from venturing outside our domain by causally connecting ornaments with one another.

III.
Higher Ornamentation

If the Meander band, broken off at right angles, be smoothed out into a curve with constantly changing direction, just as we smoothed out the zigzag line into a wavy line, then a spiral ornament will result (n must be sufficiently large, that is to say, every single spiral of the ornament consists of $(n + 1) / 8$ complete turns). One can also easily change not only a single stripe but indeed the entire square lattice of a Meander pattern into continuously connected spiral stripes. If this figure is smoothed out, the result will be an extended double spiral ornament. One finds such ornaments, veritable "spiral-orgies," for example, on ceramic work from islands in the Aegean Sea.

This figure can be modified in multiple ways. From the ordinary Meander stripes synchronous spirals emerge. But one can also run adjacent Meander right-angle corners or adjacent stripes, in opposition to one another. Then we obtain, with some adjustment, symmetrically opposed spirals, i.e., spirals with opposing rotation.

When extended far enough, the individual spirals will fill any desired portion of the plane with parallel curved lines whose direction constantly changes. This gives us the *Principle of Higher Ornamentatio*n in contrast to that considered in the foregoing section in which the basis for the *normal* rhythm was the association of equivalences. After the principle of equivalence came the principle of opposition. The equivalences could be linked to formations in one or more dimensions, but the principle remained the same. With the single spiral we have something different: a rotation is distinct from the next and all further [rotations]. We have something constantly changing which never repeats itself. But where is the ornament in this? A principle must be expressed, even if it is not so simple as that of the association of equivalences and oppositions. It is the *principle of equilibrium* between two or more contradictory things, the regular insertion of intervals which do not stand in opposition to their neighbors. Admittedly, this [principle] does not yet find very precise expression in the spiral. Here we have to balance the opposition between the individual finite (arbitrarily small) encirclement and the endless (actually only arbitrarily extended) plane. The [aforesaid] operation produces the ornament consisting of concentric, equidistant circles which we meet with frequently in primitive ornaments. No doubt the spiral ornament is an advance over this last, because with the spiral, the equilibrium follows more persistently due to the ever-expanding encirclements, which approach infinity. Instead of considering the ornament as the solution to the problem of how to achieve equilibrium between the finite (arbitrarily small) and the infinite (arbitrarily large), i.e., [the problem of] the insertion of regular intervals, it is often better to say that the ornament creates a balance between the one-dimensional and the two-dimensional, in the course of which often the difference between several one-dimensional shapes is reconciled. We have a beautiful example of this in ornaments from New Guinea: eyes, a nose, a mouth and the navel are perhaps "given" on a plank. The ornament balances these shapes one with the other and with the edge of the plank. It looks a lot like a system of solutions of a first order differential equation with various singularities, and thus makes a very "mathematical" impression. Also, a large group of algebraic curves whose basis is a system of curves with singularities are very similar in appearance.

If one considers the carved and painted curves on a plank from New Guinea, one will instinctively insert "all" the curves between them. I believe that one may speak of an equilibrium between the one-dimensional exit curves and the two-dimensional plank, and that the equilibrium of these curves upon the plane as well as that between the curves [themselves] yields an ornamental [object] as much in keeping with the artist's expressive intent as it is with the observer's sensation. The apprehension of an equilibrium between one and two dimensionality also gets support from reports by travelers in these regions of the tendency among natives to fill in every empty space on a piece of paper upon which they were told to draw the course and branching of a waterway.

Perhaps we can also make clear the difference between the way we treated ornament in this section and that in the previous section in the following manner: the description of the simplest ornaments foregrounded *arithmetic*. The simple row of points corresponds to the series of rational numbers and perhaps the triplets to the series of rational numbers with the denominator 3. Every angle in the square lattice can be designated with a pair of whole numbers, so too, the figures in the square lattice by means of aggregates of pairs of whole numbers, and so on. Just as the row of points divides up the linear continuum of the curve running around the vessel, so the curves on the plank divide up the two-dimensional continuum between the exit curves and those on the edges. But here the division of a linear continuum into equal parts does not correspond to the simplest Rhythm. To be sure, the curves follow one another somewhat uniformly, but that appearance can only be grasped mathematically in an approximative way. One can no longer describe the figure as arithmetical. One needs expressions such as continuity and equilibrium, i.e., expressions that in mathematics denote function theory.

Certain kinds of tatooing seem closely related to this kind of ornament, e.g., that of the Maoris of New Zealand. Apparently, to these people tatoos seem more beautiful if the shapes of eyes, nose and mouth are not offered to the viewer in isolation but rather balanced by a system of curves.

It is also noteworthy that one finds such ornamentation in Italian paintings from the 13th century. In these, the navel is almost always represented by a spiral, limbs are reconfigured with a system of curves having little to do with reality. The back of the hand, and the surface of the eye between the corners and the pupil, display systems of balanced curves.

Also among such figures as we have been considering in this section are analogous figures outside ornamentation which on the other hand one can unite with ornamentation on account of a causal connection between them. One often meets with the opinion that spiral ornaments hark back to the widespread use of bronze spirals as armbands. For peoples having no acquaintance with bronze, one could suppose that the ornament originated with spiral forms among plants and animals. But, quite apart from the fact that these non-ornamental spirals are mostly three-dimensional curves and only ever appear singly (1) whereas spiral ornaments generally appear in systems, it seems, as has been already emphasized, that such a relationship has no great significance.[1]

Of course, the New Guinean curve systems, the New Zealander tatooing, etc., may remind one of similar natural appearances, for example, of the curved system on the surface of a tree trunk cut in two, or the system of streaks in many minerals. For the judgment that there is a causal connection stemming from mere similarity it is important to ponder the fact that ornaments of this kind appear in the most varied places at the most various times. So it is found also on very old Chinese ceramics (Yang-shao style---2500), to cite but one example. It seems, however, that the true cause must be sought in the human intellect.

Alongside the influence of natural objects, however, another important influence also appears through a possibility, lying outside the human mind, namely, the effect of technique, or more precisely, that of technical devices. Neither the pot shaped by hand nor that shaped by the potter's wheel is a body rotated with mathematical precision. But for aesthetics, the difference plays an enormous role, because the person looking at the two vessels will have a very different impression of each. The inaccuracies of the pot thrown on the wheel lie outside the limits of the

[1] An exception are the spirals on the wings of locusts found on Mt. Kilimanjaro. [Author's footnote]

human powers of perception, while the hand-formed pot, through its irregularities, arouses the feeling in the observer that he is looking at something individual, something shaped by an individual. But one can also analyze mathematically the difference between the two pot shapes. One need only take the human as such into account in the analysis. ---The use of the potter's wheel is one of the very first steps in the direction of the mechanization of human life. Rotary movement begins its fateful influence and with it the repression of the individual in human labor.

We run across similar differences in ornamentation itself, differences in exactitude among represented forms. They are so conspicuous for anyone who looks closely at an ethnological collection of ornaments that we cannot here overlook them. What a difference there is, for example, between the ingenious yet imprecise style of the colorful carvings of the New Guineans and the less imaginative but wonderfully precise carvings of the New Zealanders, or indeed those of the Malayans. This difference is in many ways brought about by the kind of implements used in manufacture. In their carvings, the New Guineans employ only sharp stones and shells, the New Zealanders have more reliable metal tools at their disposal, and the Malayans are a highly cultivated people whose tools and hand work have attained a high level of development.

Material also plays an important role in ornamentation. The zigzag ornament above Roman portals is a typical stone ornament, rococo ornaments are typical ornaments in stucco, and many forms, for example the carvings on the Oseberg ship, are characteristic of woodwork. Obviously, forms which originally and by nature belong to one material, can be imitated in another material. --- With these exceptionally modest suggestions, in comparison to the scope of the question of the relationship between technique and ornamentation, we must here be satisfied.

IV.
Three-dimensional Ornamentation.

With most people, the impression of something decidedly spatial, three-dimensional, is bound up with a specific sense of well-being which influences that impression in many ways. And this connection appears in ornamentation also. For example, we find flat ornaments that function spatially through a particular treatment--the so-called ribbon ornaments. Two stripes intersect each other. At the intersection, the border lines of one stripe stop and those of the other continue. This second stripe seems to lie *in front of* the other. Such ribbon ornaments are especially beautifully developed among many African peoples who portray very complicated spatially entwined ribbons. Also in Africa, one finds ornaments not just spatially portrayed but actually realized in space, e.g., seats or thrones made out of two circular plates. The top plate is supported all the way round by a row of statuettes, sometimes upright and sometimes on their heads. Here, then, we can speak of a genuine spatial rhythm, a relationship between equivalence and opposition realized in three dimensions.

Among the much admired negro sculptures the masks interest us particularly here whose representation of the human head is but the occasion to romp around in space. Here, a large domain of shapes made up of curved surfaces is held in rhythmic organization and raised to a high point of development.

We find very remarkable spatial ornaments among the inhabitants of an island near New Guinea, alternately referred to as New Ireland and New Mecklenburg.[2] Their carvings, pink [lit. "reddish-white"] in color, catch the eye of every visitor to an ethnological collection: a largish

[2] Translator's note: these are separated islands in my atlas; I don't know what Dehn is referring to by Grösse Seelands. Maybe a geological period.

tree or portion of a tree trunk, hollowed out by the work of generations without the use of metal tools, and carved up into such a maze of figures, connecting rods, and surfaces as to be almost impossible for the viewer to take in at a glance. Certainly, many parts of this large-scale work possess a symbolic meaning. These play no role in our investigation. We notice in these creations a reveling in the possibility of embedding things within each other in space, together with an instinct for rhythmic organization, the repetition of equals and opposites.

From these wild and fanciful creations, in which the rendering of all sorts of natural objects and animal symbols almost conceal the rhythmical, mathematical aspects, we pass on to absolutely pure spatial ornaments with nothing but associations of equivalence manifesting the realm of spatial possibilities. To begin with, the five regular solids (polyhedra) are indeed as much ornaments as are the Meander examples or other noteworthy analyses of the square lattice, as long as we do not designate as ornament mere embellishment but also every realization of a system governed by principles. Those examples, just as much as the regular solids, set forth mathematical theorems. Human play and experiment surely discovered most of these figures. One among them, the dodecahedron, possibly was found in nature and recognized as ornament, i.e., its rhythm was understood. With the ones already known, that would have completed knowledge of the group of regular solids. The fact that outside these five solids no realization of even the simplest spatial rhythms is possible, cannot be understood without a mathematical proof, as is the case for every assertion concerning unlimited possibilities. The consideration of this assertion and its proof does not belong in the analysis of ornamentation.

It is astounding that inventive humans discovered this rhythm. The division of the circle into equal parts may seem self-evident, but the division of the surface of a sphere into equal parts--and such divisions involve the regular solids even if the ancient Greeks were not aware of it from the start--these are still today a marvel. The forms of the regular solids and the existence of irrational proportions between lines, something very real and something totally abstract, were the two main engines driving the development of classical Greek mathematics.

The Etruscans, who depicted especially beautiful solutions of the square lattice, must also have been aware of the dodecahedron. One of the Egyptian pyramids employs the form of the octahedron, and the pyramids are to be designated as ornaments gigantic in size, if we give the word "ornament" a more general definition. These pyramids have square bases and their apexes lie directly over the midpoint of their bases, and so the edges of the sides are of equal length. However, at least on some pyramids, the edges of the sides are also equal to the edges of the base, i.e., we have in these before us approximately half of an octahedron. Now the pyramids have their effect in the first instance not through their proportions, through their precise or approximate equivalences, but through their absolute size and venerable age. One can, if one wishes, also analyze this size mathematically but, as already touched on earlier, one must again analyze at the same time the human factors involved. A wall seems high not only when its height exceeds that of a man or the height from which he can jump down without injury, but rather when it is so high that [its top] cannot be reached with the usual means of steps and ladders found in ordinary houses. Should a height, occasionally seen, exceed that of exceptionally tall buildings, such as churches, then it seems monstrously high, if it is a building. A rock face 150 m. high, which is the height of the pyramids, does not make such a majestic impression, because it is not as all isolated like the pyramids, because it is not so intangibly smooth and because one has the almost constant impression that there must be another side from which one can get to the top of it.

We have made this attempt at an analysis because size plays an inordinate role in all edifices, and because we want here to put forward something about its relationship to ornamentation. No doubt the facade of large churches, say, the facade of Notre Dame in Paris, or that of the Strassburg Cathedral, is a two-dimensional ornament, but its dimensions have a great share in its effect. The same is true for the strongly rhythmic facades of the great edifices of antiquity and for the gothic churches whose organization of architectural elements--the enduring system of central and side aisles and the arches connecting them we admire. The same holds true for their spires or for the strong stereoscopic effect in the halls of Genovese palaces. In more elegant reduction the effect of elevation is lost. One need only think of the reproduction in alabaster of the leaning tower of Pisa with the candle shining through the pasted-on red window. It is different though with photography or especially pictorial representations of buildings because we are accustomed to project the two-dimensional projection back into the space and real situation, so that the original magnitude takes effect.

In landscapes, we have the pure impact of spatiality, i.e., the three-dimensional, together with the impression of magnitude. Only color, particularly color contrast can increase the effect. This contrast is the only aspect of ornament, of the system of equivalences and oppositions, still remaining to discuss. We think perhaps of the view of the south side of the Mont Blanc from the heights on the adjacent side, this world of large and small summits and peaks behind and above one another, with the surfaces of the glacier in between, rising from the river valley in the depths far above eye level, this world takes its effect through the spatial variety, the size and the sharp contrast between the subdued colors of the stones, the dazzling ice and the deep blue of the heavens. Here there is hardly available any sensational appeal to the mathematical in humans.

We can also speak of the *multidimensionality in music*. One dimension delivers time, the other pitch. Thus emerges a series of sounds, a melody, played by an instrument, a curve in a two-dimensional field. If then a second melody comes in, played by a different instrument, a three-dimensional effect results, similar to that with ribbon ornaments, when the pitch of one voice lies now over, now under, that of the other. One says--and many have the analogous impression--that both voices entwine as from time to time the deeper voice appears to lie behind the higher one. The same holds true, of course, for more than two voices, as perhaps four voices performing a chorale. As to the conformity, the rules, that the members of such a multidimensional complex homogeneous and create out of them, in our sense of the term, an ornament, we cannot speak here. But we still wish to discuss a different connection with our study of ornamentation. The *accompaniment* with many correlated sound series of differing pitches often gives the impression of something *spatially extended*. The solo voice in this case traces its course over this surface of the accompaniment as background. Such an accompaniment effecting an impression of extension we may compare with that which we called higher ornamentation. In both structures the impression is evoked of something extended, two-dimensional, by means of what is one-dimensional--in the one case through a system of constantly changing curves, and in the other case through many correlated series of sounds. I have the impression that in contemporary music sometimes this rhythm, which consists in the two-dimensional accompaniment and the solo voice, disappears, and then the entire sound fabric appears to be a single multidimensional continuum, in which of course the sound volume can still play a role as a dimension.

V.
Overview of the Various Kinds and Principles of Ornamentation.

Hopefully, the developments in the foregoing three sections will have made our approach clear. This approach is also applicable to the numerous branches of ornamentation which we have not considered until now. In the first place, we think of the richly diverse field of *textiles*, understood in the widest sense of the term: knotted carpets, especially from the Middle East, woven work from every corner of the ancient and modern world, Egyptian, Persian, Chinese embroidery, appliqué and inlay and, finally, in Western countries, lacework. Every technique gives rise to specific ornaments which however all organize themselves according to the governing principles of ornamentation which are generally the same for all techniques. The first is the widespread *principle of interlacing*: in Section III we became acquainted with a method of covering a surface with curves. But there exists a completely different method of performing this task. Starting from any given curve, one branches out at appropriate locations, i.e., one extends from there different curves, "tendrils," with the same initial tangent. In the same fashion, one proceeds to add more "tendrils" to these new curves, and so on. In this way, one may also achieve a solution in two dimensions. It is above all, in respect to the principle, a more arbitrary method [of covering a surface with curves] than that considered above. As one knows, however, it can be harmonious and graceful when it is produced with a certain tact. Perhaps one can discover with more attentive inquiry, beyond the evident simple symmetries, still finer rhythms significant for the effect in such an interlaced ornament. We will only mention in passing that groups of open spaces, e.g., in lacework, can play a major part in the ornamental effect.

We want to go a bit deeper into another principle, especially because it has also great importance for acoustical ornamentation. This principle can be known immediately from seeing large knotted carpets, perhaps like those found in the Victoria and Albert Museum in London, a few larger natural objects stylized as ornaments, looking rather like images of insects, are strewn over the large surface in parallel courses. Every one of these figures, perhaps twenty in number and each more or less different from the next, stands out in sharp relief from the surrounding area. These figures are connected by a lattice of curves which themselves are further decorated with small units, for example rose blossoms. Thus, we have here the principle of *different classes of units* combined into a larger unity. Smaller units are joined and these combinations further integrate with a greater unity. This greater unity can be compared to that which we call a *theme* in music. Like the principal figure in a carpet, the theme in a movement of a sonata, a quartet, or a symphony presents itself in many variations. One could also compare the principal figure in a carpet to a melody and its variations. But the difference here is that the carpet figure is much more articulated than a theme. *One* specified unity is taken as a the simplest basis and from that emerge all the variations by enrichment (which one may perhaps compare to interlacing) and by resolution. Smaller units in the carpet, of which we were speaking above, e.g., rose blossoms, correspond to *figures* in pieces of music which continually recur within a movement and thereby contribute to its coherence. Thus we have another association between acoustical and visual ornamentation.

One finds single figures, units in combination, that not only dominate a single work, but are also characteristic for an entire historical period or for a large country. Thus a specific ear-shaped curve is characteristic of rococo, just as a row of vases connected by festoons is characteristic of the Empire style, etc. The ornamentation on almost all ancient American work is characterized by pairs of parallel straight lines which change direction with short bends. One observes this stylistic element also in imagistic works, such as depictions of gods or mythical beasts. Ones finds it besides among the coastal inhabitants of Northeast Asia and sometimes Indian sculpture, too, will remind one of it.

VI.
Limitations of the Study of Ornament

In conclusion, only one more suggestion regarding where ornamentation, in pictorial art, overlooks its proper domain. In numerous works of essentially ornamental character, in those such as we have here discussed, we encounter representations from nature. Time and again, imitations of nature get used for ornamental purposes. We have often emphasized that the main source for ornamentation is to be looked for in human nature itself. But sometimes nevertheless one has the feeling that a work originates with an external stimulus and its resonance in mankind, and so it is when one looks at the New-Guinean dance plank which consists of two birds of paradise arranged in symmetrical opposition to each another and whose tail feathers exhibit a system of spiral curves very much of the sort we called solution-, or equivalence types in Section III. Occasionally, one would also like to think about a certain pleasurable sensation when a person associates his own rhythm with the representation of a natural object, when, as it were, he feels he imposes his own rhythm upon nature. However, the domain of ornamentation is abandoned if the person seeks in the first place to represent, if ornament is subordinated to representation, if, one could say with some exaggeration, perspectival depiction is placed above symmetrical arrangement.

Completely realistic representation of a natural object is alway a task impossible to fulfill. Alone the ever changing appearance presented by a person, animal, or other natural object offered to the observer, is already not renderable by a single representation. Now, it is often the case that the image of a simple curve, namely parts of contour lines, appear to afford in certain characteristic aspects something *essential* for representation, as for example in Egyptian representations the profiles of the head together with the frontal outline of the trunk, etc. As children people become quickly accustomed to accept these curve systems as representing humans and recognize from the specific forms of these curves even distinct persons. However, there are things in nature for which what is essential consists of something that resists a simple representation. Such are above all natural objects for which the essential is an impression of infinitely differing things, e.g., the surface of the sea with infinitely many waves and patches of light, a large tree with infinitely many leaves, a field of grain, a mass of hair on the head, and many others. There is a good description of the helplessness of a young painter confronted with such a problem in Gottfried Keller's novel, "Green Henry." There, the hero, already adroit at drawing outlines and shading, wants to draw a tree, a large tree with a variously ramifying trunk and a prodigious quantity of leaves. Despairing at last, he draws a slender little tree with distinct branching and modest leafage. To solve this problem, that of representing an apparently infinite multiplicity, ornamentation can be of help. Everywhere in so-called archaic art hair on people's heads, or an animal's fur coat, is represented by means of ornaments, e.g., by a system of small units that cover a plane in regular arrangement in which the head of hair or the pelt-covered animal body is presented. These small units are perhaps spirals or systems of concentric circles made to designate ringlets, or they are simple lines extending from a point at the top of the head. Right at the beginning of our investigation of ornaments we saw that such regularly-ordered units were associated with the impression of something that can be continued arbitrarily, and thus something connected with the impression of infinity.

In more developed art, a freer, higher ornamentation takes the place of these simple basic elements in which, for example, the "infinite" hair is represented by a sequence of strands which are arranged in space by a system of analysis. We noted above in section III that such an arrangement automatically induces the observer to receive sympathetically the images which afford a continuous passage between the represented images, i.e., between the contours of the strands. Thus ornamentation helps in two very different ways with the representation of a multiplicity of [appearances involving] infinitely many things.

Different principles must be acknowledged in the representation of different natural objects, e.g., in the drawing of masses of leaves, a gradation according to size: small units will be irregularly combined with larger ones that represent the amount of leaves [respectively] on twigs and on branches.

We cannot here enter into a discussion of how similar principles turn up in the representation of moving water, fields of grain, forests, crowds, etc. We have already transgressed the bounds of ornamentation and ornament here is but a servant. We want only to refer to a strange appearance. There are carved planks from New Guinea (e.g., in the British Museum and in the Völker Museum in Stuttgart) which on first appearance exhibit a jumble of tendrils and holes ungoverned by any higher principle. Gradually, however, it occurs to the observer that here, too, the attempt was made to depict something infinite, namely, the infinite diversity of vegetation in the primeval forest. The perception is strengthened by the animals one finds portrayed on some planks. Perhaps we have here also a passage from ornamentation to veritable representation, indeed to the representation of something of infinite variety.

We set before us once again the domain of ornament: the product of human work since the earliest times, spread across the whole world, evident in vastly many shapes, in the capitals of gigantic Egyptian columns, in the scorned wares of European factories, in clothing and jewelry, in the ever-repeated pious utterances gathered as wall decoration on mosques, on book spines and on weapons, on almost everything used in daily life. But among all these forms, only a few principles must be acknowledged, basic tendencies of the human spirit which are inextricably bound up with music, architecture, and every artistic expression. These principles are mathematical in nature and mathematical propositions are discernible in many ornaments. Thus we are able through our observations to find a connection between the beautiful works of those who create sensuous forms and the manifold shapes with which the mathematician works in the abstract.

[Translation by Jon T. Ording. Note on some key terms:
Ornamentik refers to the subject of study, Ornament to the figures studied, *das Ornamentische* to the quality of being an ornament, "ornamental". The term *Rhythmik* seems to refer to the individual's characteristic, quasi-analytical, action in observing and representing external objects ornamentally.]