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The Topology of Roman Mosaic Mazes

Anthony Phillips

ABSTRACT

The 46 Roman mosaic mazes that are well preserved and/or well documented enough for the maze-path to be intelligible fall into 25 distinct topological types, which in turn can be built up out of seven elementary submazes. This understanding allows the theoretical reconstruction of some of the mazes that have been partially destroyed and signals instances of faulty restoration in some of those that appear more or less intact today.

To the memory of my father,
John Goldsmith Phillips,
1907–1992

Floor mosaics were a common form of interior decoration in Roman times, and a maze pattern was a perennial theme: the remains of some 57 maze-patterned Roman mosaic floors have been discovered, scattered over the territory of the Roman Empire from England to North Africa, and with assigned dates ranging from the first century B.C. to the fourth century A.D.

These mazes were catalogued in 1977 by Wiktor Daszewski, whose work *La Mosaïque de Thésée* [1] is my fundamental reference, along with Hermann Kern's encyclopedic *Labyrinthe* [2], which contains, in addition to all of those documented by Daszewski, another five items, [K:124a, 124b, 126, 132a, 162]. In general, I will refer to each mosaic maze both by its number in Daszewski's catalogue [D#*m*] and its figure number [K:*n*] in Kern's work.

Most of these 57 mosaic floors are still accessible today, although some have been wrecked and others were never more than fragments. In particular, there are five cases ([D#14, K:142], [D#35, K:152], [D#37, K:156], [D#54, K:165], [D#59, K:163]) in which a maze was dug up, drawn and then lost or destroyed; [D#6] has disappeared without being drawn; two English mazes [K:124a, 124b] were drawn and are now covered over; an Algerian maze [D#3, K:130] was photographed but its current state was unknown to my references; two mazes [D#5,13] are known or suspected to exist but information about them is incomplete or unavailable.

In this article we will study the 46 mazes that are well preserved and/or well documented enough in the two references for the maze-path to be intelligible. (There are cases in which ambiguities might have been resolved by access to better photographs or to the mazes themselves; I hope to return to these cases in the future with better documentation.) In particular we will examine the *topology* of these mazes, those properties that depend only on the relative position of the elements of the maze and are independent of whether the maze is large, small, round, square, and so forth. The idea of a classification of maze-path types goes back to Daszewski; the work presented here may be viewed as an elaboration and quantification of his work.

Because these mazes are unicursal (without bifurcation), the maze paths themselves have no interesting topology. But these mazes have the following additional property: the path lies on a certain number of distinct levels. We will consider two mazes to be topologically equivalent if one can be changed to the other by a *level-preserving* deformation. Then we will find that these mazes fall into 25 distinct topological types, which in turn can be built up out of seven elementary submazes. This understanding will allow us to reconstruct, on paper at least, some of the mazes that have been partially destroyed, and to detect instances of faulty restoration in some of those that appear more or less intact today.

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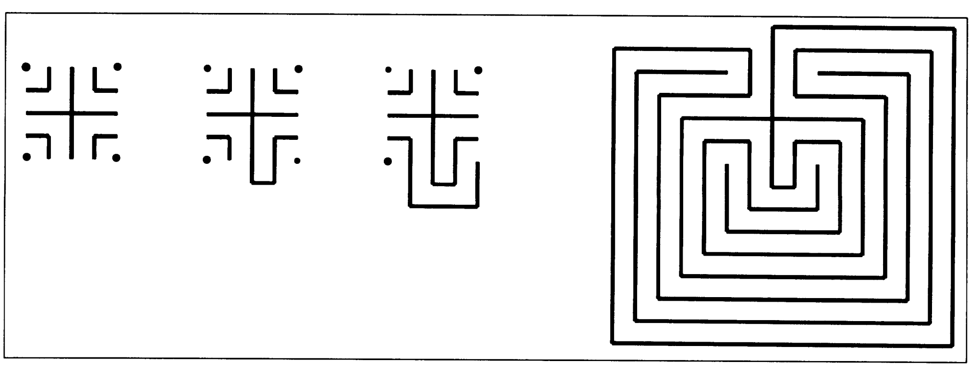


Fig. 1. The Cretan maze drawn from a nucleus consisting of a cross, four L's and four dots. Beginning at the bottom, each free end is joined, around the bottom, to the next free end on the opposite side.

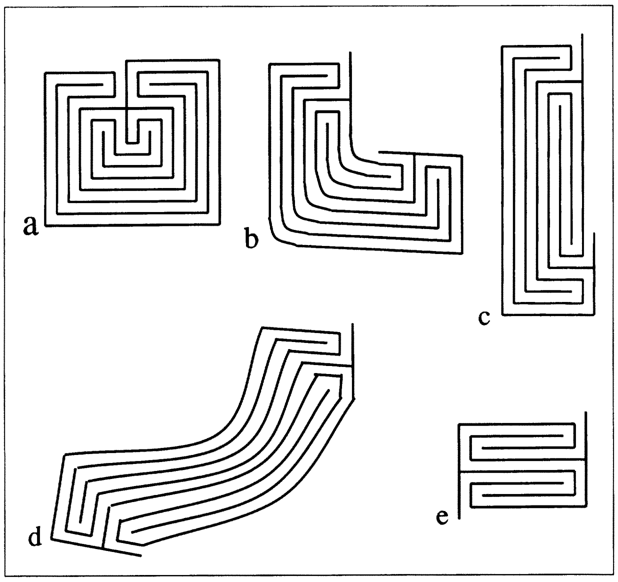


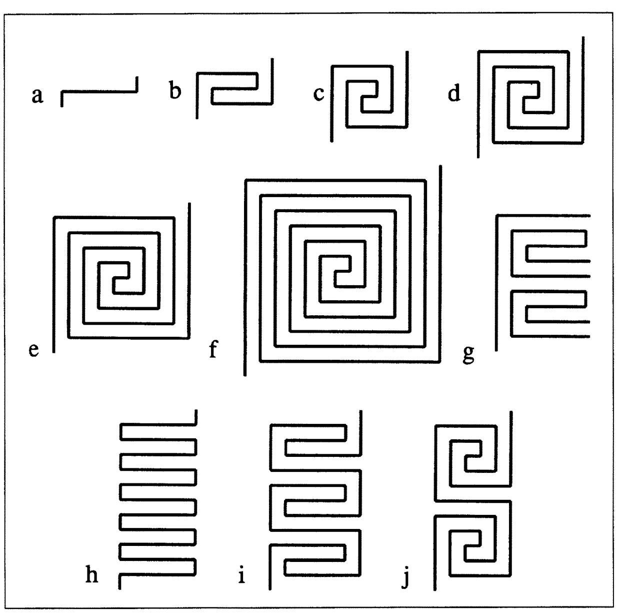
Fig. 2. The Cretan maze (a) is split down its axis and deformed through stages (b), (c), (d) to its unrolled form (e).

The Cretan Maze

The simplest [D#49, K:119] and the oldest [D#62, K:160] mosaic mazes extant today are already essentially perfect forms. We have no record of the work that went into developing this concept, but there is a related class of figures going much farther back in history: various forms of the famous Cretan maze.

The earliest illustrations of this maze that can be securely dated are a pair of figures [K:102] on a clay pot from Tell Rifa'at, Syria (dated before 1200 B.C.), and a doodle [K:104] scratched on the back of a clay accounting tablet in King Nestor's palace in Pylos (Western Greece) and hardened by fire when the palace burned down around 1200 B.C. According to Kern, the scratchmarks in the second example strongly suggest that this maze was drawn from a nucleus as seen in Fig. 1 (see [K:6]). The game of drawing this nucleus and completing it to a maze is still played today [3]. For future reference, note that if we number the levels in the maze-path beginning with zero on the outside and ending with 8 in the center, the thread through the maze will traverse these layers in the order 0 3 2 1 4 7 6 5 8. It turns out that this *level sequence* completely determines the topology of the maze, in the following sense. If we consider just the class of mazes that share with this design the following three properties: (1) there is a single path that runs from the outside to the center; (2) the maze is organized on concentric levels that we may number from zero on the outside to n, the *depth* of the maze, in the center, and the path fills out the levels one at a time; (3) the path changes direction whenever it changes level; then any maze with level sequence 0 3 2 1 4 7 6 5 8 can be deformed, without changing the relative position of the walls, to coincide with the figure from Nestor's palace or its mirror image. These mazes may be called *simple alternating transit mazes,* or SAT mazes. Their structure is analyzed more systematically in other sources [4].

Fig. 3. Elementary meander mazes and how they are stacked. In this figure each maze is represented by its maze-path; the walls are not shown. (a) γ_2; (b) γ_4; (c) γ_6; (d) γ_8; (e) γ_{10}; (f) γ_{14}; (g) α_{10}; (h) γ_2^6; (i) γ_4^3; (j) γ_6^2.

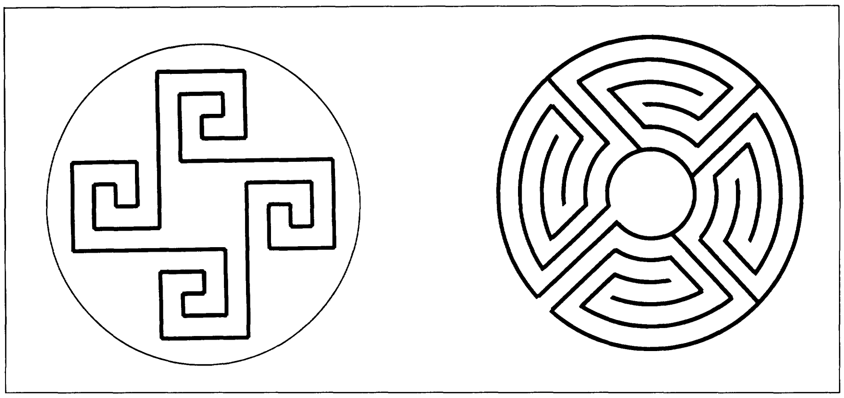


The form of this maze has come to be called the Cretan Maze because of its association with Crete and with the legend of the Minotaur. It appears on coins minted for the Cretan city of Knossos [K:52–58], dating from 500 B.C. down to Roman times. This form was known to the Etruscans [K:112] and to the Romans, as can be seen by its appearance in graffiti on the walls of Pompeii [K:107,108] and in two of our mosaics ([D#17, K:147] and [K:126]).

The coins of Knossos bear at least two other designs relevant to this study. One [K:50] is the four-level maze with level sequence 0 3 2 1 4. It appears on a coin dated circa 431–350 B.C. and is evidence that the Cretans had gone beyond the labyrinth game to analyze the structure of the Cretan maze, because in fact the Cretan maze can be realized as two copies of 0 3 2 1 4, one nested inside the other. This example strongly suggests that the principle of combining mazes by unrolling (Fig. 2) and stacking (Fig. 3) was already understood at the time this coin was made. Otherwise it is difficult to understand how the second, smaller maze and its symbolic interchangeability with the first could have been discovered, since simplifying the game itself leads to the trivial maze 0 1 2 3 4, as in [K:6C]. As we shall see, this fact was certainly understood by the Roman mazemakers; the example points to an earlier understanding. Once the mazes are unrolled, it becomes obvious that they are intimately related to, and perhaps derived from, the meander patterns ubiquitous in primitive decoration. It should be noted that meanders and mazes appear interchangeably on the earliest coins from Knossos: a maze, a swastika-meander, a meander-frame (see below) all may occur, but never more than one form on any given coin, suggesting strongly that the three were equivalent symbols for the Labyrinth. Above all, it is remarkable that (see Fig. 3) *all the mazes occurring in antiquity are meander mazes.* (Since I will be arguing that many of the mazes have lost their original form, this needs to be clarified: every Roman mosaic maze that has survived intact is a meander maze, and I will make the case that the mazes that do not appear to be of this type fail to do so because of faulty restoration or recording.)

A second relevant design from Cretan coins [K:45, 46, 49] is shown on the left of Fig. 4. Kern calls it a 'swastika-meander'. This figure hints at the possibility that the linking of several SAT mazes into a larger, cyclic configuration, which as we shall see is the standard organizing principle for Roman mosaic mazes, was also of Cretan origin. This swastika-meander may in fact be read as the maze-path of

Fig. 4. (left) The 'swastika-meander' from a Cretan coin (reversed for comparison), with (right) the plan of the Roman mosaic in Avenches, Switzerland. The meander gives the exact path through the maze, except for the entrance and exit corridors.



four unrolled 0 3 2 1 4 mazes linked in a circle, almost exactly the plan of the Roman mosaic maze at Avenches, Switzerland [K:119]; only the entrance and exit corridors are missing.

THE 'STANDARD SCHEME'

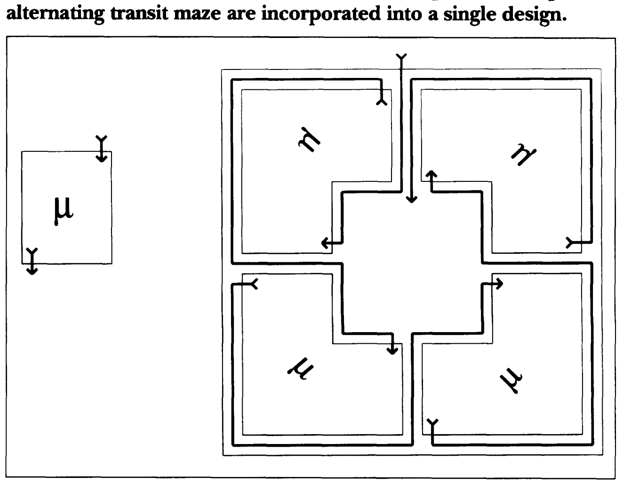
Whether or not the Romans inherited this organizing principle from the Cretans, it seems that they came up with the idea of adding an extra corridor to give the maze-path two ends—one outside, one inside—and of choosing the repeated submaze from a wide range of SAT mazes. Cretan echoes include the number of copies used (still almost always four), the more-or-less symbolic representation of the fight between Theseus and the Minotaur (a frequent central element), and the typical framing of the entire maze by a crenelated wall with towers and gates, the line of the crenelation recalling the meander-frames that occur as yet another labyrinthine element on the coins of Knossos [K:42, 47, 48]. This *standard scheme,* which occurs in 42 of our 46 examples, has the submazes arranged radially around the central area (Fig. 5), so that their entrances are towards the center. The maze-path enters midway along one side, runs up to the wall enclosing the center, turns and enters the first of the submazes. After traversing the first maze, the path runs back towards the center, to the top of the second, and so forth. After traversing the last maze, the path runs again back to the center, and this time enters it. The overall path may run clockwise or counterclockwise. This scheme is explained by Daszewski [5]. The large maze at Mactar, Tunisia [D#57, K:143], has a variant appropriate for its semicircular, two-sector design.

Metaphorical explanations for this design, beyond its explicit connection with the Theseus myth, lead one into fascinating speculation [6]. Its decorative potential, however, is clear. The bulk of the floor area is covered with black (usually) and white square tiles laid in straight lines, with (usually) the freeform polychrome work reserved for the center. Most of this mosaic was therefore no more expensive to lay than the checkerboard pattern in a bathroom floor. But thanks to the maze design, this large black-and-white area can be read for an intricate, global meaning. (There is a small set of particularly lavish examples, [D#8, K:137], [D#11, K:136], [D#43, K:141], in which the walls or the path are indicated by a multicolored braid.) The division of the maze into consecutive sectors gives the plan an overall intelligibility, besides forcing the path into a large-scale meandering motion superimposed on that of the submazes themselves. That the division into four quadrants is arbitrary is attested by the existence of an eight-sector example [D#50, K:132] and a three-sector one [D#60, K:133]. Clearly four is most natural when the overall shape of the maze is rectangular (in fact the eight-sector maze is round, and the three-sector hexagonal) but the Romans do seem to have preferred four sectors (the round, labyrinthiform decorative relief [K:113a] and the other eight round mazes, [D#8, K:137], [D#9, K:139], [D#14, K:142], [D#18, K:120], [D#44, K:161], [D#49, K:119], [D#56, K:131], [K:162], all have four sectors, while [D#57, K:143], mentioned above, is a semi-circular maze with two sectors).

THE POMPEIAN VARIATION

In the standard scheme with four sectors, the overall maze design had almost perfect four-fold rotational symmetry. It is only 'almost' perfect because the side holding the entrance gate has two paths running towards the center (the path going from the entrance to the top of sector 1, and the path from the bottom of sector 4 to the center), whereas each of the other three sides only has one. The maze-plan of the fountain [K:114] has a similar problem. An ingenious solution was found in a series of Italian mazes of the first centuries B.C. and A.D.—[D#25, K:151], [D#28, K:129], [D#30, K:153], and [D#36, K:155] (House VIII 2 16, shown in Fig. 6)—and in my reconstruction [D#35, K:152]. The earliest are in Pompeii. The first three sectors are arranged as in the standard scheme, but the fourth sector contains a different submaze, one in which the path enters *and exits* through the top [7]. This submaze was constructed so that along its edges it almost exactly matches the maze it replaces, so on cursory inspection the entire design seems exactly symmetrical. It is important to understand the contribution

Fig. 5. The 'standard scheme' in which four copies of a simple alternating transit maze are incorporated into a single design.



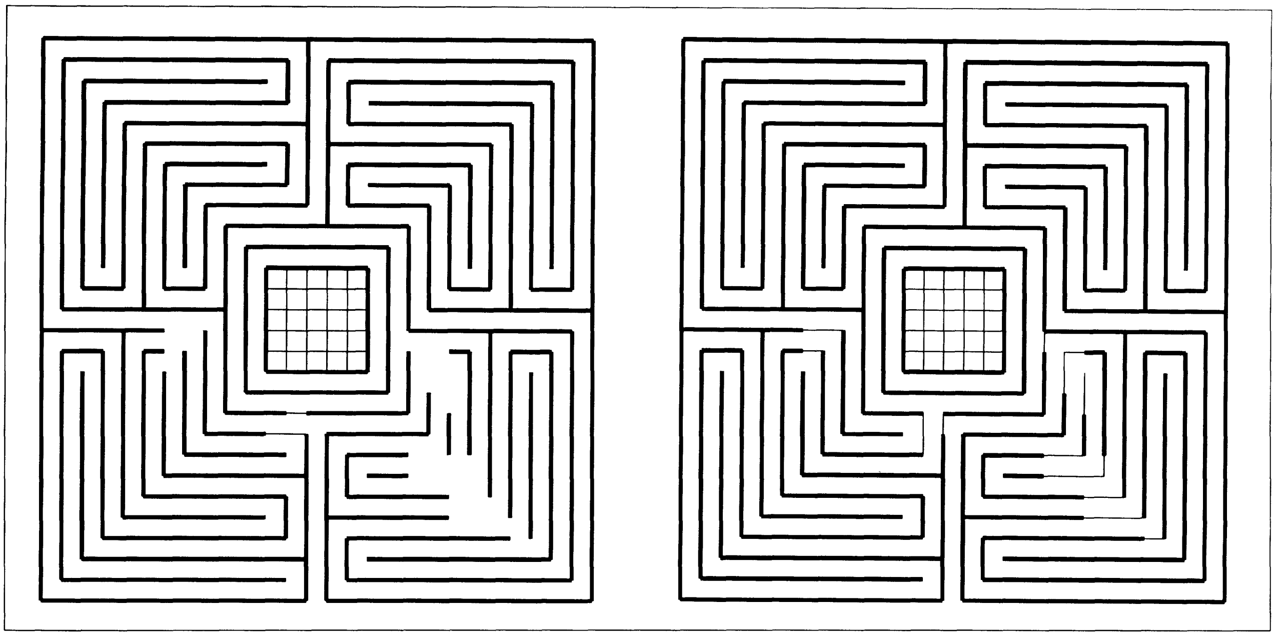


Fig. 6. Pompeii, House VIII 2 16: (left) current state and (right) proposed reconstruction.

of the artist/engineer who devised this alternative topology. This was not an impromptu modification of just one maze. Not only was it copied in several examples, but the theoretical principle involved was used to symmetrize two different designs (and, very likely, a third). (See my analysis of the maze in the Villa di Diomede [D#35] below.)

Every intact Roman mosaic maze follows the standard scheme or its Pompeian variation. The mazes we know of that do not follow this scheme have all been lost or damaged, and I will argue that they only fail to follow the scheme because of incorrect repairs, in the case of those damaged, or incorrect recording, in the case of those lost. There are four exceptions: two Cretan-type mazes [D#17, K:147] and [K:126], the great double maze at Pula [D#61, K:158] and one of the two six-level mazes in [K:162], which seems in fact to have been incorrectly laid.

Table 1. Frequency of submazes occurring in standard-scheme Roman mosaic mazes. Not counted here are two mosaic mazes of pure Cretan type [D#17, K:147], [K:126] and the irregular double maze at Pula [D#61, K:158].

sub-maze	depth	number of examples
γ_2^2	4	1
γ_4	4	3
γ_2^3	6	1
γ_6	6	1
γ_2^4	8	2
γ_4^2	8	14
γ_{10}	10	1
γ_4^3	12	6
γ_6^2	12	1
$\gamma_6\gamma_8$	14	1
γ_{14}	14	1
γ_2^8	16	2
γ_4^4	16	4
γ_6^3	18	1
γ_4^5	20	2
α_{10}^2	20	1
Total		43

Decomposition into Elementary Mazes

The submazes in Roman mosaic mazes can be analyzed further into elementary mazes, which are composed into larger mazes by the same nesting operation that makes 0 3 2 1 4 7 6 5 8 out of two 0 3 2 1 4's. In the unrolled form, nesting becomes stacking, where the maze $\mu\mu'$ is formed by identifying the bottom layer of μ with the top (zeroth) layer of μ'. The elementary mazes that occur are shown in Fig. 3: these are the meander mazes γ_2 = 0 1 2, γ_4 = 0 3 2 1 4, γ_6 = 0 5 2 3 4 1 6, γ_8 = 0 7 2 5 4 3 6 1 8, γ_{10} = 0 9 2 7 4 5 6 3 8 1 10, γ_{14} = 0 13 2 11 4 9 6 7 8 5 10 3 12 1 14 (γ_{12} does not occur in this corpus) and α_{10} = 0 9 6 7 8 5 2 3 4 1 10. (The maze γ_2 is not elementary in the same way as the others, since it can be written as $\varepsilon\varepsilon$, or ε^2, where ε is the one-level maze 0 1; in this corpus, however, ε always appears to an even power, and the γ_2 notation is more appropriate in this context.) In Daszewski's classification, powers of γ_2 are described as *serpentins*, powers of γ_4 (and also α_{10}) as *en méandres*, and the higher γ's as *en spirale*.

Table 1 enumerates the different submazes that occur, with their frequency; the five Italian mazes mentioned above are represented by their regular sectors. Not listed in this table are the two occurrences mentioned above of the Cretan maze itself, a round one [K:126] and a unique rectangular one [D#17, K:147]. Note in this table that the unrolled form of the Cretan maze, i.e. γ_4^2 = 0 3 2 1 4 7 6 5 8, is by far the most frequent, and that the various powers of the Cretan and half-Cretan maze (γ_4 = 0 3 2 1 4) account for 25 of the 39 examples. This fact underlines the intellectual continuity between the mazes on the coins of Knossos and these mosaic examples.

We may now describe the Pompeian variation more explicitly. When the first three sectors have $\gamma_4^2 = (0\ 3\ 2\ 1\ 4)(0\ 3\ 2\ 1\ 4) = 0\ 3\ 2\ 1\ 4\ 7\ 6\ 5\ 8$, the standard fourth sector can be written as 0 3 2 1 4 7 6 5 8 −1, the 8 −1 representing the path into the center, while the Pompeian variation has 0 1 4 5 8 7 6 3 2 −1: the path is a γ_2^2 traced down and back, performed so to speak in augmentation. When the first three sectors have γ_4^3 and the standard fourth sector is 0 3 2 1 4 7 6 5 8 11 10 9 12 −1, the variation has 0 1 4 5 8 9 12 11 10 7 6 3 2 −1 (a doubled γ_2^3). If in fact the maze in the Villa di Diomede [D#35, K:152] was of this type, its fourth sector was 0 1 4 5 8 9 12 13 16 15 14 11 10 7 6 3 2 −1, and not the $\gamma_4^4 = 0\ 3\ 2\ 1\ 4\ 7\ 6\ 5\ 8\ 11\ 10\ 9\ 12\ 15\ 14\ 13\ 16$ shown in the picture.

Table 2 analyzes the corpus of known Roman mosaic mazes with legible maze-paths. I give for each maze the topological type of the path: $4 \times \gamma_4^2$ means a standard scheme with four linked copies of 0 3 2 1 4 7 6 5 8, etc. The Pompeian variation is signalled by an asterisk (*). (Note that [D#57, K:143] is not quite standard, see the remarks above.) I also indicate whether, according to my analysis, there have been errors made in restoration, or in the recording of mazes that no longer exist. Each of the erroneous mazes will be analyzed in the next section.

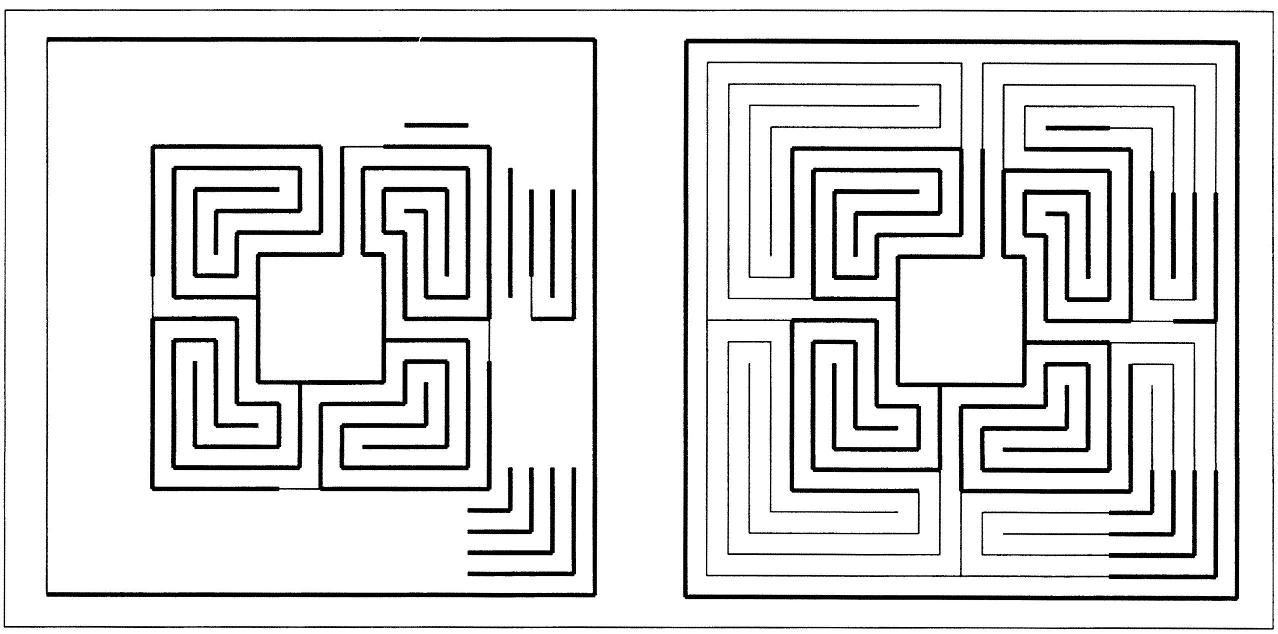
Table 2. The 46 Roman mosaic mazes of this study, in their order from Daszewski's catalogue [13], along with their figure number in Kern's book [14]. Each is listed with its topological type, and indicates whether errors have been made in execution, restoration, representation or interpretation.

D#	K:	Type	Errors?	D#	K:	Type	Errors?
1	118	$4 \times \gamma_4^2$	yes	37	156	$4 \times \gamma_4^2(*)$	
2	123	$4 \times \gamma_4^2$		41	167	$4 \times \gamma_6^2$	yes
3	130	$4 \times \gamma_{14}$		43	141	$4 \times \gamma_4^2$	
4	116	$4 \times \gamma_{10}$		44	161	$4 \times \gamma_2^8$	yes
7	171	$4 \times \gamma_4^3$	yes	46	125	$4 \times \gamma_4^3$	yes
8	137	$4 \times \gamma_4$		47	127	$4 \times \gamma_4^2$	
9	139	$4 \times \gamma_2^2$		49	119	$4 \times \gamma_4$	
11	136	$4 \times \gamma_4^2$		50	132	$8 \times \gamma_2^4$	yes
14	142	$4 \times \gamma_4^2$		51	148	$4 \times \gamma_4^3$	yes
15	124	$4 \times \gamma_4^2$	yes	52	135	$4 \times \gamma_2^8$	yes
16	157	$4 \times \gamma_4^2$		53	170	$4 \times \gamma_4^4$	
17	147	γ_4^2		54	165	$4 \times \alpha_{10}^2$	
18	120	$4 \times \gamma_4^2$		55	169	$4 \times \gamma_4^5$	
21	122	$4 \times \gamma_4^3$	yes	56	131	$4 \times \gamma_4^3$	
22	140	$4 \times \gamma_4^3$		57	143	$2 \times \gamma_4^5$	
24	121	$4 \times \gamma_4^4$		59	163	$4 \times \gamma_6^3$	
25	151	$4 \times \gamma_4^2(*)$		60	133	$3 \times \gamma_4^2$	yes
28	129	$4 \times \gamma_4^2(*)$		61	158	nonstandard	
29	134	$4 \times \gamma_6\gamma_8$		62	160	$4 \times \gamma_4^4$	
30	153	$4 \times \gamma_4^3(*)$			124*a*	$4 \times \gamma_6$	
34	149	$4 \times \gamma_4^2$	yes		124*b*	$4 \times \gamma_4^2$	
35	152	$4 \times \gamma_4^4(*)$	yes		126	γ_4^2	
36	155	$4 \times \gamma_4^2(*)$	yes		162	$4 \times \gamma_2^3$	yes

Errors in Execution, Restoration, Representation or Interpretation

Mosaics, while made of relatively indestructible material, are quite fragile; they share these qualities with pottery, but whereas the shards of a pot can usually be reassembled in only one way, once tiles get scrambled the pattern is lost. Several of the mazes in the form we know them today contain gross errors in the path layout. An entire sector may be inaccessible, or the maze may lack an entrance, an exit, or both. Since these were large, beautiful works, whose execution implies both artistic talent and engineering ability, it is very unlikely that this was their original condition. Daszewski gives a fascinating account of an ancient repair in the great Kata Paphos maze [D#8, K:137], noticeable on close observation because it occurred in the central pictorial

Fig. 7. Chusclan: (left) current state and (right) proposed reconstruction. Elements to be added to or removed from the current state are drawn in with fine lines.



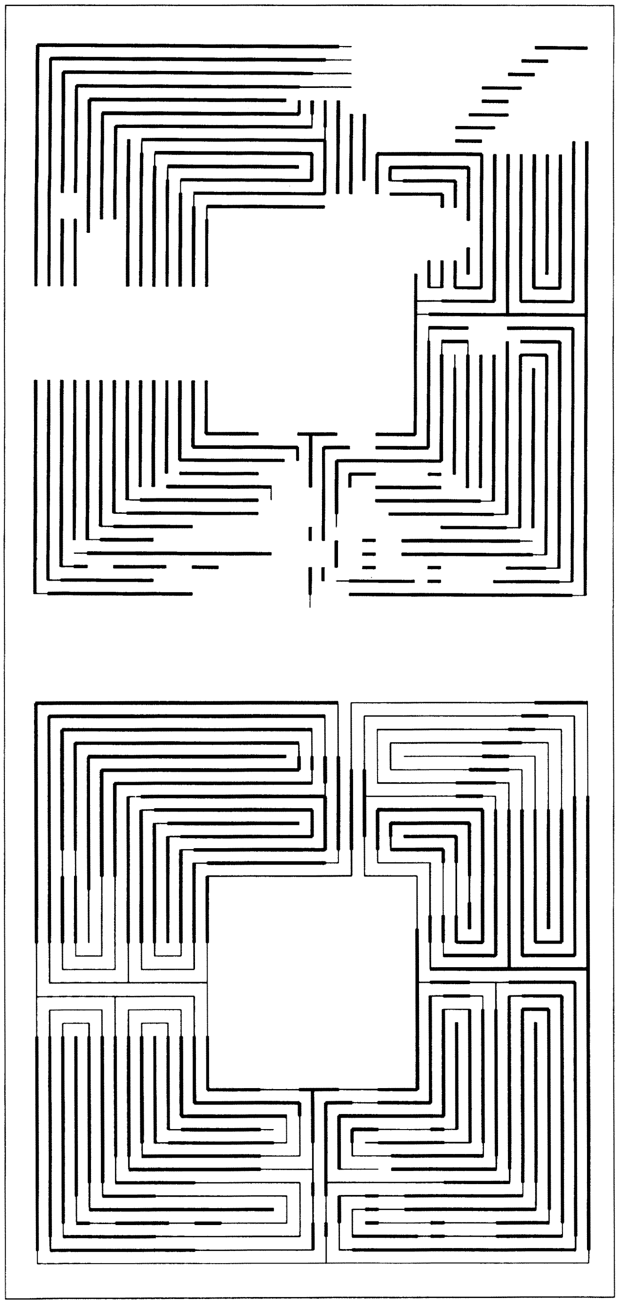


Fig. 8. Syracuse: (top) current state and (bottom) proposed reconstruction. The γ_6 submaze occurs in only one other standard-scheme maze. Elements to be added to or removed from the current state are drawn in with fine lines.

area: the repairers had not exactly matched the style of the original; deeper investigation revealed a corresponding perturbation of the substrate. Another old repair is mentioned by Tite in an 1855 article in *Archaeologia* [8]. How much less reliable are repairs made after two millennia, when perhaps one-half of the pattern is missing and what is left is scrambled around the edges! Furthermore it is very likely that the excavators who sketched their finds had to work by extrapolation from part of the pattern; this would explain some of the errors that we find in the drawings.

The rest of this article treats separately the 14 mazes for which I believe that errors occurred in execution (one case), in restoration, in representation or in interpretation, plus additional remarks on mazes [D#50] and [D#61].

D#1 Annaba, Algeria

Daszewski presents a drawing [1], plate 40a ([K:118]), of a rectangular maze of type $4 \times \gamma_2^4$, with the path interestingly adapted to the unusual nonsquare shape. An obvious error in sector 3 leads the path into the outer wall when it should double back on itself. It is not clear from my sources whether this is an error in the drawing or in the maze itself.

D#7 Vienna

This is a lavish polychrome square maze of type $4 \times \gamma_4^3$, illuminated by four mosaic pictures: in the center, Theseus fights the Minotaur; on the left Ariadne gives him the ball of thread; on top they embark; and on the right the abandoned princess sits on her rock. This maze is in excellent condition and correctly reproduced and analyzed in both my sources. The error I would like to signal is elsewhere. Matthews [9] reproduces a drawing of this maze in which there are four paths issuing from the Naxos picture: one leading to the center, one over to where Ariadne is shown giving Theseus the thread, and the other two leading back to each other. Similarly Theseus has four options. The maze has been ingeniously turned into a puzzle. The culprit for this substitution is presumably Matthews's source Georg Friedrich Creuzer, who admits to having abbreviated (*abgekurzt*) a larger representation of the maze [10]. In fact, besides the incongruity of these choices with what the pictures represent, a puzzle involving choices is totally alien to the spirit of mazes from antiquity. The mazes that have come down to us from prehistory, antiquity and, for that matter, the middle ages are all unicursal. What the ancients enjoyed in these mazes must have been akin to what people still enjoy in optical illusions. The paths are too complicated for the eye to follow, and yet they are right there on the page (or the floor) where the finger or the foot can trace them through. A nice modern example of this phenomenon is on the cover (and on p. 73) of Minsky and Papert's book *Perceptrons* [11].

D#15 Chusclan, Gard, France

The drawing, Daszewski [1], plate 58a ([K:124]), shows a square maze inside a wall with towers on each of three sides. Daszewski states that about one fourth of the mosaic is missing; the loss seems to be all in the maze, which is shown with five inner levels forming a $4 \times \gamma_4$, and bits of wall suggesting five more outer levels. This is all very unconvincing, especially since the entrance to the 'inner' $4 \times \gamma_4$ faces the one wall without a tower. The maze was surely a $4 \times \gamma_4^2$ (Fig. 7).

D#21 Caerleon, England

Plate 43 in Daszewski [1] ([K:122]) is a carefully executed drawing of a square mosaic maze. It shows about 40% of the maze as missing, and includes a hypothetical reconstruction of part of the center area. What is shown looks like sectors 2 and 3 of a clockwise $4 \times \gamma_4^3$ except that in sector 2 the path runs 0 3 2 1 6 5 4 7 8 11 10 9 12 instead of γ_4^3 = 0 3 2 1 4 7 6 5 8 11 10 9 12. This is very unlikely, since sector 3 has the second, standard form. Changing sector 2 to a γ_4^3 involves resetting a handful of tiles in an area close to the missing portion of the maze.

D#34 Ostia

Plate 47 in Daszewski [1] ([K:149]) shows a photograph of part of a square mosaic maze. Sectors 1 and 2 are shown

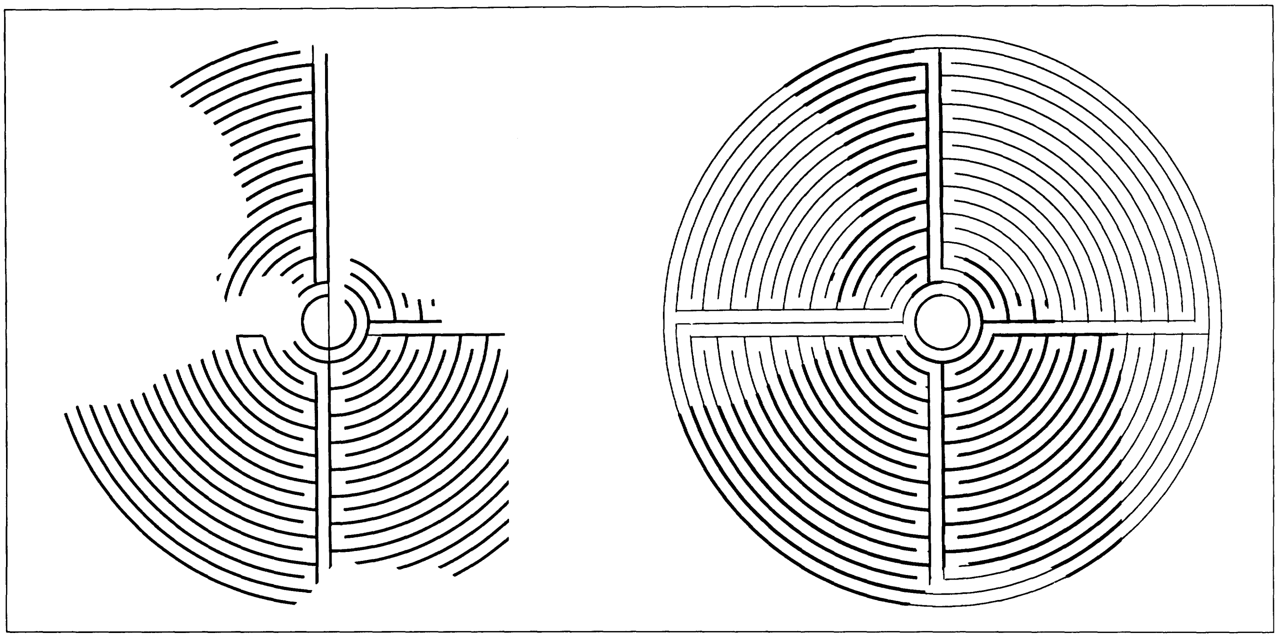


Fig. 9. Sabratha: (left) current state and (right) proposed reconstruction. Elements to be added to or removed from the current state are drawn in with fine lines.

along with half of sectors 3 and 4; the maze is a clockwise $4 \times \gamma_4^2$ with moderate damage, clumsily repaired. The restoration has perturbed the maze-path in sector 1, introducing two T's where the path would branch. The pattern may be corrected by a small reshuffling of tiles.

D#35 Pompeii, Villa di Diomede

This maze is only known to us through a drawing (Daszewski [1] plate 48a, [K:152]) published in 1841. The drawing shows a square maze of type $4 \times \gamma_4^4$, except that the four sectors are linked each to the next, counterclockwise, with no entrance from the outside nor exit into the center. The date assigned to this maze (80–60 B.C.) agrees with those of two other Pompeian mazes ([D#30, K:153] and [D#36, K:155]), which are of type $4 \times \gamma_4^3$ (*) and $4 \times \gamma_4^2$ (*), respectively. (The other Pompeian $4 \times \gamma_4^2$ (*) [D#37, K:156] has a later date.) This makes it very plausible that the original design was a $4 \times \gamma_4^4$ (*), with a fourth sector entered and exited through the top.

Why would this elaborate drawing have been made incorrectly? There are some internal clues. The mosaic as shown has suspiciously exact four-fold symmetry, down to the windows on the towers along the edges, which are drawn as rigorously identical from one side to the next. This suggests that much of the maze was missing when it first was excavated and that the artist copied one sector, saw that the sectors were similar and made them all exactly the same. Further evidence of damage is the blank center, incongruous with the elaborate treatment of the wall surrounding the maze. Most likely the center had earlier been torn from the maze to be marketed as a separate work of art, like the isolated polychrome Minotauromachies catalogued as [D#12, K:150], [D#58], [D#19, K:115], [D#26, K:145], [D#27, K:146], [D#31, K:144] and [D#45, K:168].

D#36 Pompeii, House VIII 2 16

Daszewski's plate 48b [1] shows a photograph ([K:155]) of a square mosaic maze. The maze was originally of type $4 \times \gamma_4^2$ (*), as evidenced by the single radial path on each side and by the diagnostic single non-nested radial segment to the left of the entrance (see [D#25, K:151], [D#28, K:129] and [D#37, K:156], where the entire design appears). A damaged area at the top of sector 4 was incorrectly restored: the entrance path bifurcates and sector 4 is a dead end. Rearranging a few tiles in the damaged area recreates the correct form. The path ends in a loop encircling the center, an unusual feature (see Fig. 6). Daszewski [1] gives this maze "*4 secteurs de méandres*" whereas his usual characterization of Pompeian-type mazes is "*3 secteurs de méandres et un de type serpentin*".

D#41 Syracuse/Taormina

This maze is documented in the sources only by the drawing Daszewski [1], plate 51 ([K:167]) of its ruins. It is badly damaged; in particular it now has 13 levels on the left and right but only 12 on the top and bottom. Measurements of the drawing show that the central area (almost always square) is exactly one row too tall, and the existing maze overall one too short. This suggests adding an extra level at the top of the center, and another at the bottom of the maze, as in Fig. 8. Furthermore, portions that seem unlikely to have been modern inventions strongly suggest that the maze was of type $4 \times \gamma_6^2$. This is a 13-level configuration with the compound nesting shown in sectors 1, 3 and 4 of the ruin. The γ_6 submaze is rare but it occurs in another insular Italian maze: [D#28, K:134] on Giannutri, and also, rather inexplicably, by itself, adjacent to a large, elaborate geometric mosaic floor in a villa in Halstock, Dorset [12].

D#44 Sabratha, Lybia

The photograph, Daszewski [1], plate 56a ([K:161]), shows a circular maze, badly damaged (Fig. 9). The submazes are clearly of type γ_2^8, but exactly how they were connected is difficult to determine. Most likely the entire center has been incorrectly restored: the central medallion should not be bisected by a wall, and the frame about the medallion is really a loop ending the maze-path. A similar loop runs around the outside of the maze, and probably joined the beginning of the path; such loops are somewhat unusual, but occur also in the great Kata Paphos maze [D#8, K:137], in the Pompeian [D#36, K:155] and in the maze at Pula (see below).

D#46 Coimbra, Portugal (Museu Monografico)

This is a square maze, of type $4 \times \gamma_4^3$ (Fig. 10). The photograph reproduced as Daszewski [1] plate 40a shows that about 30% is missing: the corners of sectors 1 and 2 and the outer three levels along their common side. A small error just below the central field leads the path from the entrance directly across to sector 2. There also seems to be another error at the very edge of the photograph, creating a dead end in sector 2. Each of these errors can be corrected by repositioning a few tiles. The photograph suggests that a trench was dug at some time right across the middle of the maze. If so, it is amazing that these were the only errors produced.

D#50 Cormerod, Switzerland

There is no real error here, just a faked photograph. In Daszewski's plate 31a the central medallion has been rotated 90°, no doubt to give a more pleasing picture of the maze. The true configuration is shown in Daszewski [1] plate 32 and in [K:132].

D#51 Orbe, Switzerland

Daszewski presents a drawing [1], plate 58a ([K:148]), as well as a partial photograph (plate 58b). The maze is square, of type $4 \times \gamma_4^3$; the drawing shows an error in sector 1: the path from the entrance to the top of the sector is missing, and the sector begins in a dead end. The photograph does not show that part of the maze, but hints at considerable damage in the hidden area. It is likely that the error is in or near the damaged area and can be charged to faulty restoration or perhaps overgeneralization on the part of the artist who made the drawing.

D#52 Henchir el Faouar, Tunisia

The drawing, Daszewski [1], plate 18a ([K:135]), shows a square maze of type $4 \times \gamma_2^8$ set counterclockwise; a small error at the bottom of sector 2 leads that sector to a dead end and reopens the maze to the outside. The photograph (Daszewski [1], plate 18b) reveals that the error was made by the artist in reinventing an area that had been destroyed.

D#60 Gamzigrad, Yugoslavia

This is a hexagonal maze of type $3 \times \gamma_4^2$, with no entrance into the maze nor exit into the center. Kern states that this is surely an error, and in this I believe he is mistaken. The maze is drawn so as to look like three faces of a cube seen from above one vertex, with one submaze occupying each face and the central field reduced to the width of the path, occupying the vertex. Surely the focus of this design was the effect of perspective and the perfect three-fold symmetry of the configuration; entrances and exits would have destroyed this. The labyrinthiform decorative element from Side, Turkey, [K:113a] is a round $4 \times \gamma_4^2$ with similarly perfect (here four-fold) rotational symmetry.

K:162 St-Cyr-sur-Mer

This mosaic is unique in this corpus in that it has two separate mazes. They are both of type $4 \times \gamma_2^3$ and appear as part of a large design incorporating a variety of decorative elements. One of the mazes, the left-hand one in [K:162], has an error: the path is not led back to the center after traversing the fourth sector. The photograph suggests that this was an error in execution. We can hypothesize that the right-hand design is the original, and that the left-hand one was executed by an apprentice as an (imperfect) copy.

The Great Double Maze at Pula

The maze at Pula, Yugoslavia [D#61, K:158] deserves special mention. This is undoubtedly the most intricate of all Roman mosaic mazes. About one fourth of the maze is destroyed, but there is enough information left to allow the reconstruction (see Fig. 11) of the entire design (with the exception of an ambiguity near the entrance, which a better photograph might clear up). Although it has the overall appearance of the four-sector standard scheme (with an additional circuit of the entire maze and one of the center, like the Kata Paphos maze [D#8, K:138]), this maze clearly transcends the genre. It is in fact a double maze, where the

Fig. 10. Coimbra (Museu Monografico): (left) current state and (right) proposed reconstruction. Elements to be added to or removed from the current state are drawn in with fine lines.

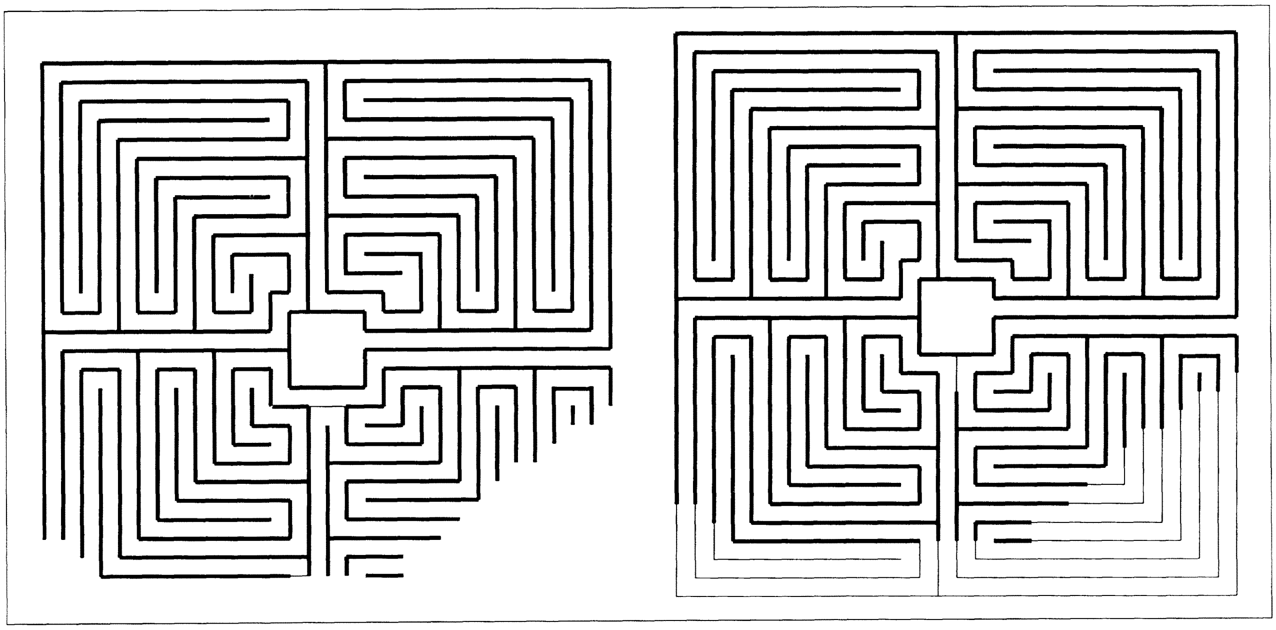
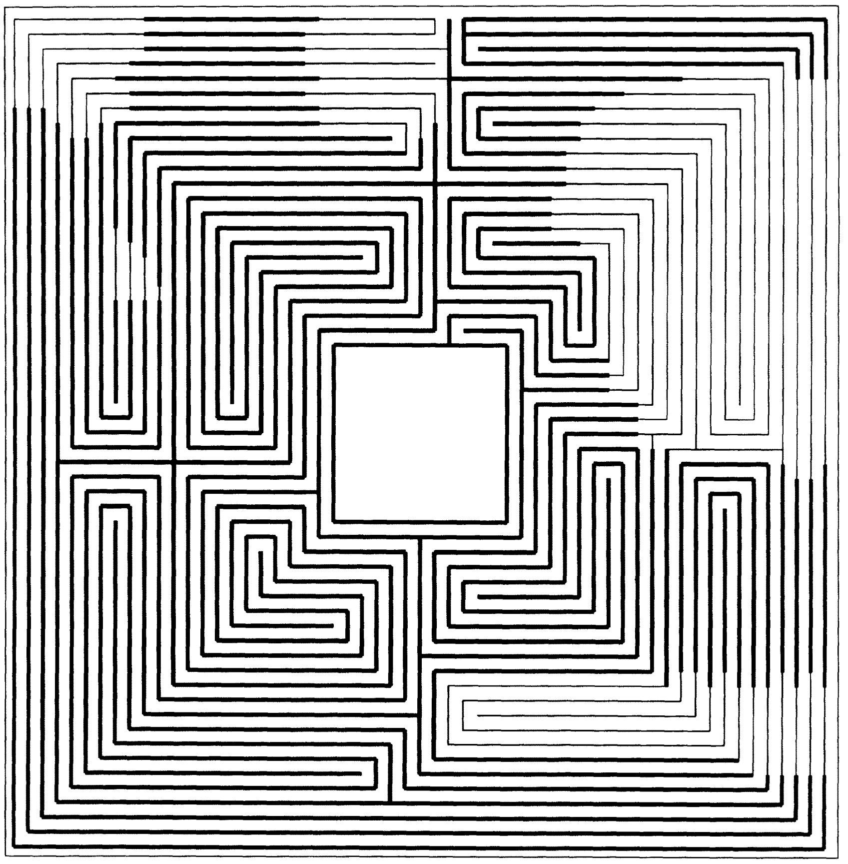


Fig. 11. Pula: proposed reconstruction. Elements to be added to the current state are drawn in with fine lines.



path runs from the outside to the center *and back out again* (by a different route). This can easily be seen from examination of the central area, which is intact. Numbering the sectors from 1 to 4 clockwise from the entrance, they are visited in the order 4 1 2 3 2 3 4 3 2 1. Other features include embedded meander mazes (a γ_8 and a γ_{10} in sector 4, a γ_6 in sector 1) and a mock doubled $\gamma_4\gamma_6$ in sector 3.

Conclusion

Devising a unicursal maze to fill out an area in an interesting and symmetrical way is a topological problem as well as an aesthetic problem. Given constraints on size and overall organization, there is only a small number of topologically distinct solutions; this has allowed our mathematical close reading of Roman mosaic mazes, an analysis of which could probably be extended to the rich corpus of mazes in Medieval manuscripts, architecture and works of art. Each of the solutions that occur was first discovered by someone, somewhere; a fascinating element of the study of ancient mazes is the contact with these ingenious, unsung topologists of the past.

References and Notes

1. Wiktor A. Daszewski, *La Mosaïque de Thésée. Etudes sur les mosaïques avec représentations du labyrinthe, de Thésée et du Minotaure (Nea Paphos II)* (Warsaw: PWN-Editions Scientifiques de Pologne, 1977). Daszewski catalogued all of the mazes that he knew and assigned a number to each one (alphabetically by country). Some of the mazes are illustrated; the illustrations appear separately and in a different order than in the catalogue. Daszewski includes tables to instruct the reader how to go back and forth from the catalogue to the illustrations.

2. Hermann Kern, *Labyrinthe,* 2nd Ed. (Munich: Prestel-Verlag, 1983). This is an expanded version of *Labirinti. Forme e interpretazioni. 5000 anni di presenza di un archetipo* (Milan: Feltrinelli, 1981). Kern lists mazes alphabetically by location, although only the illustrations are numbered. Kern includes illustrations all of the mazes discussed in this article except for one [D#58].

3. Jean-Louis Bourgeois, private communication.

4. For a determination of which permutations of 0, 1, . . . , *n* are the level sequences of SAT mazes, and for a consideration of the problem of counting all possible SAT mazes with *n* levels, see Anthony Phillips, *Simple Alternating Transit Mazes* (forthcoming). See also A. Phillips, "La topologia dei labirinti", in M. Emmer, ed., *L'occhio di Horus: Itinerari nell'imaginario matematico* (Rome: Istituto della Enciclopedia Italiana, 1989) pp. 57–67; M. Emmer, *Labyrinths,* 16mm color-sound film from the series *Art and Mathematics,* 27 min (Rome: Film 7 International, 1984).

5. See Daszewski [1] p. 51; see also Daszewski's analysis of [D#24].

6. See Kern [2] p. 114. See also Penelope Reed Doob, *The Idea of the Labyrinth from Classical Antiquity through the Middle Ages* (Ithaca, NY, and London: Cornell Univ. Press, 1990).

7. This is explained in Daszewski [1] p. 82.

8. William Tite, "An Account of the Discovery of a Tesselated Pavement, 10th February 1854, under the Vaults of the South-Eastern Area of the Late Excise Office", *Archaeologia* **36** (1855) pp. 203–213.

9. W. H. Matthews, *Mazes and Labyrinths, a General Account of their History and Developments* (London: Longmans, Green, 1922). Reissued (Detroit, MI: Singing Tree Press, 1969; New York: Dover, 1970).

10. Georg Friedrich Creuzer, *Abbildungen zu Friedrich Creuzers Symbolik und Mythologie der alter Völker* (Leipzig and Darmstadt: Heyer und Leske, 1819) p. 29.

11. Marvin Minsky and Seymour Papert, *Perceptrons* (Cambridge, MA: MIT Press, 1969).

12. Anne Rainey, *Mosaics in Ancient Britain* (Totowa, NJ: Rowman and Littlefield, 1973) p. 87.

13. See Daszewski [1].

14. See Kern [2].